Discrete Geodesic Parallel Coordinates

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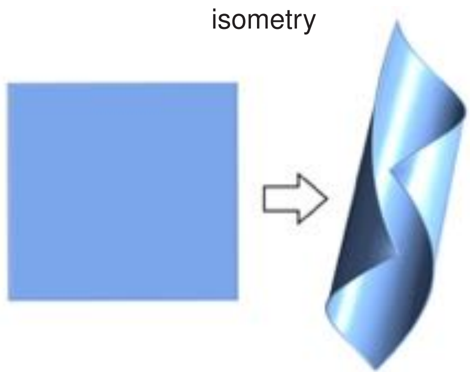
SIGGRAPH ASIA 2019 BRISBANE

KAUST
大连理工大学 DALIAN UNIVERSITY OF TECHNOLOGY
TU WIEN TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

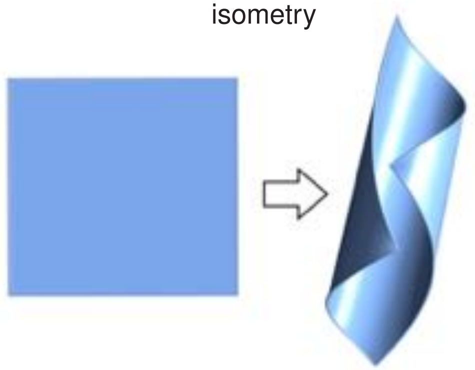
Motivation

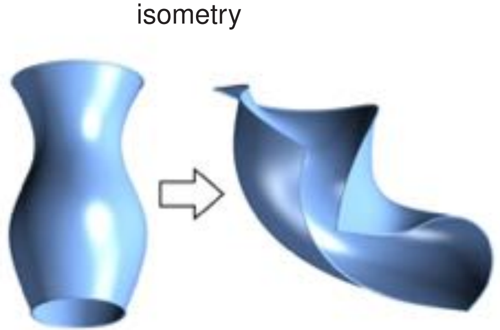
Walt Disney Concert Hall
Frank O. Gehry

Motivation



Motivation





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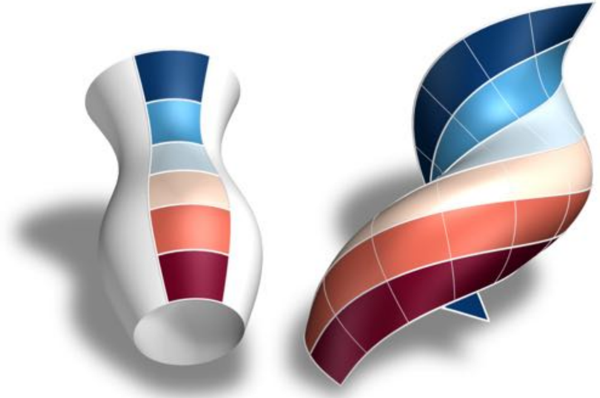
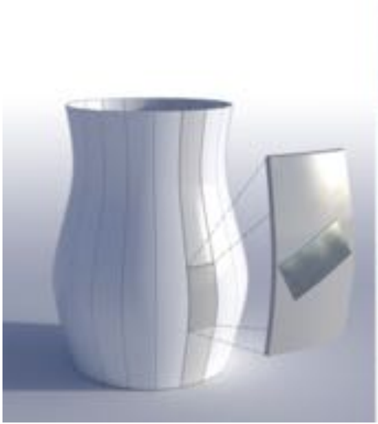


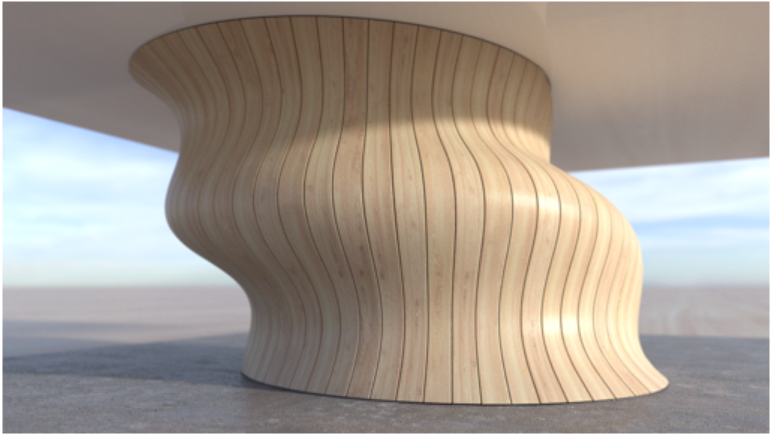
$$\text{molds} \approx \sqrt{\text{panels}}$$

Motivation



Motivation

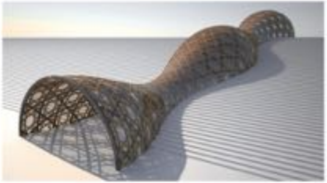
Motivation



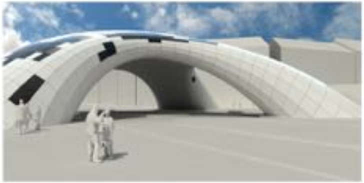
straight congruent flat strips

Related Work

[Rabinovich et al. 2018]

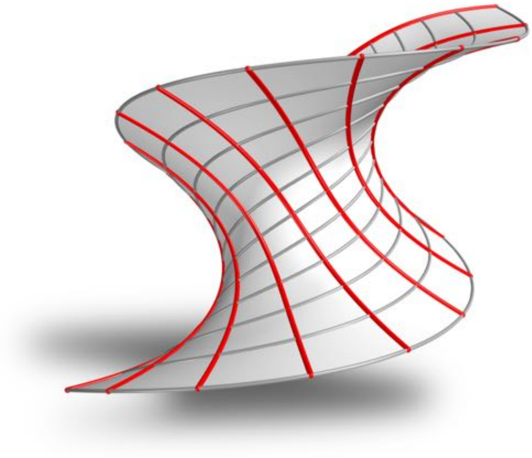


[Pottmann et al. 2010]



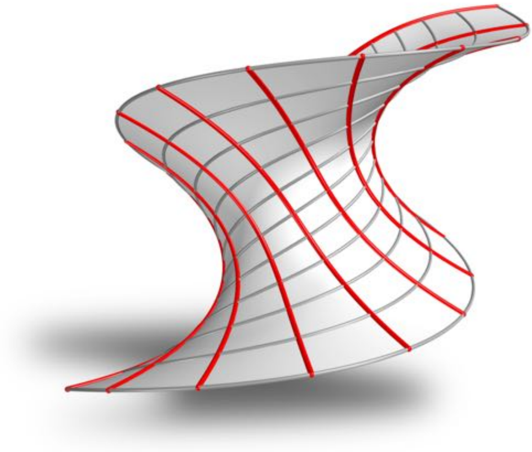
[Tang et al. 2014]

Geodesic parallel coordinates



- Orthogonal parameter lines
- The parameter lines of one family are geodesics (red)

Geodesic parallel coordinates

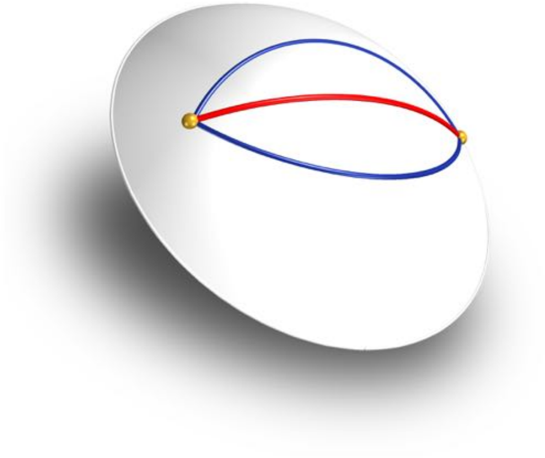


- Orthogonal parameter lines
- The parameter lines of one family are geodesics (red)

$\Downarrow$

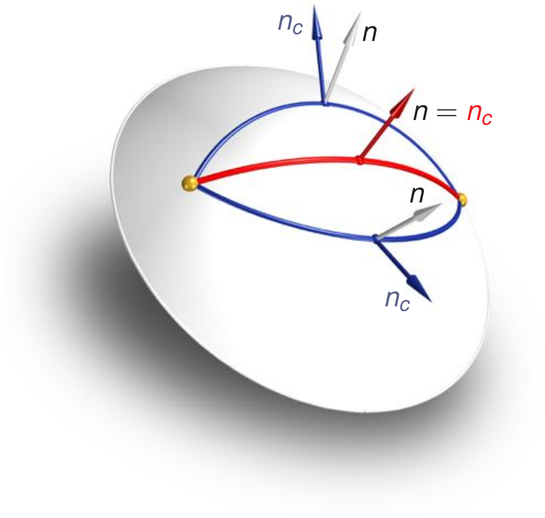
- The parameter lines of the other family are 'parallel' (gray)

Geodesic curves



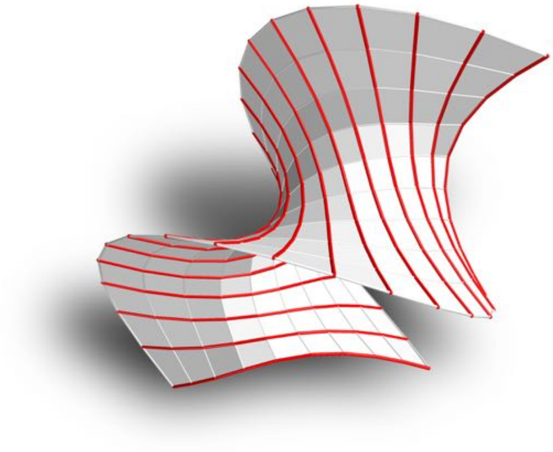
- (Locally) shortest paths on surfaces

Geodesic curves



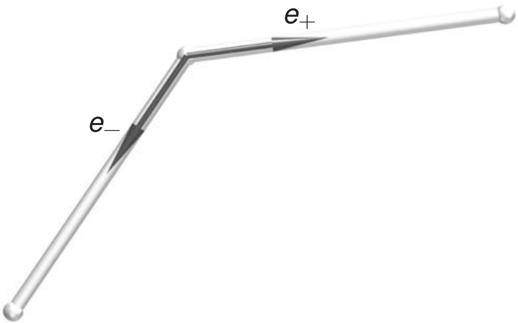
- (Locally) shortest paths on surfaces
- Principal curve normal n_c and surface normal n coincide

Discrete geodesic parallel coordinates

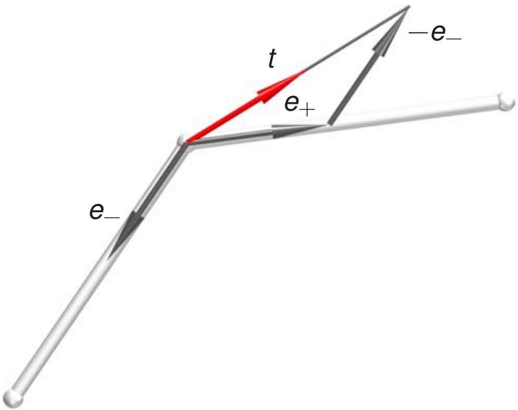


- Discrete orthogonal mesh polylines
- One family of polylines are discrete geodesics (red)

Discrete curves

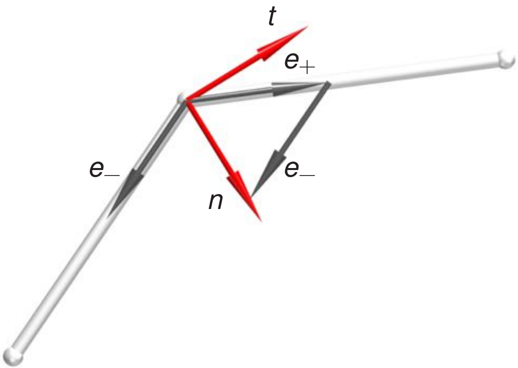


Discrete curves



$$t = (e_+ - e_-)/\|e_+ - e_-\|$$

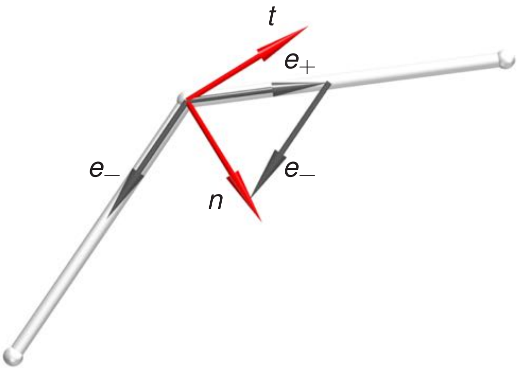
Discrete curves



$$t = (e_+ - e_-)/\|e_+ - e_-\|$$

$$n = (e_+ + e_-)/\|e_+ + e_-\|$$

Discrete curves

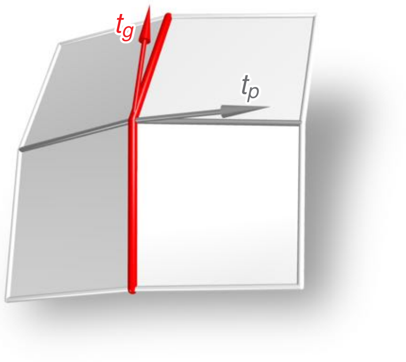


$$t = (e_+ - e_-)/\|e_+ - e_-\|$$

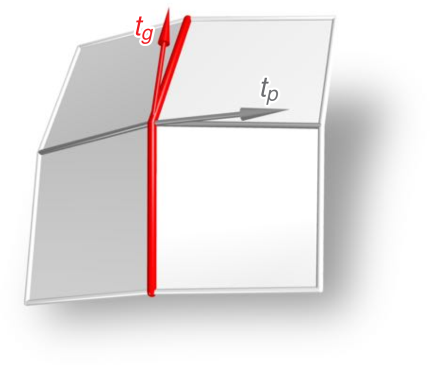
$$n = (e_+ + e_-)/\|e_+ + e_-\|$$

$$t \perp n$$

Discrete geodesic parallel coordinates

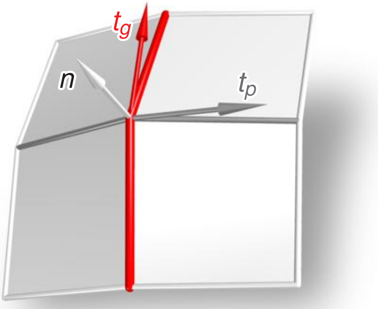


Discrete geodesic parallel coordinates



Geodesic:

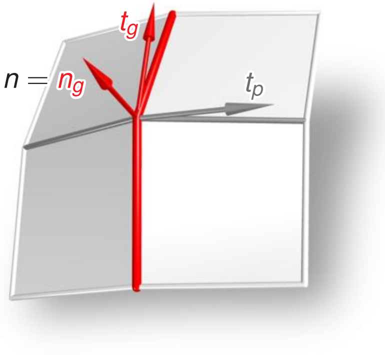
Discrete geodesic parallel coordinates



Geodesic:

surface normal: $n = t_p \times t_g$

Discrete geodesic parallel coordinates

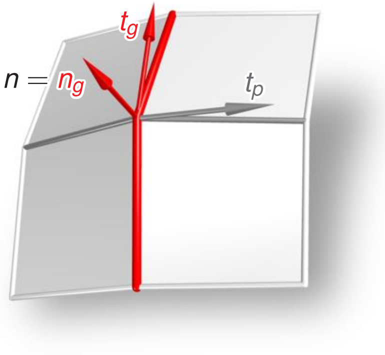


Geodesic:

surface normal: $n = t_p \times t_g$

$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$

Discrete geodesic parallel coordinates



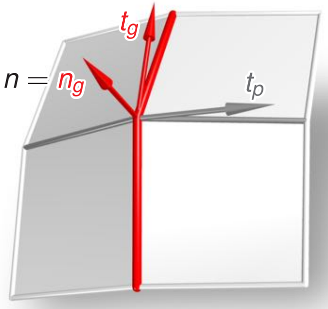
Geodesic:

surface normal: $n = t_p \times t_g$

$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$

$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$

Discrete geodesic parallel coordinates



Geodesic:

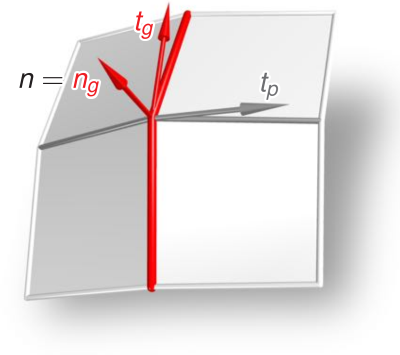
surface normal: $n = t_p \times t_g$

$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$

$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$

$= \langle e_+ + e_-, e_+ - e_- \rangle$

Discrete geodesic parallel coordinates



Geodesic:

surface normal: $n = t_p \times t_g$

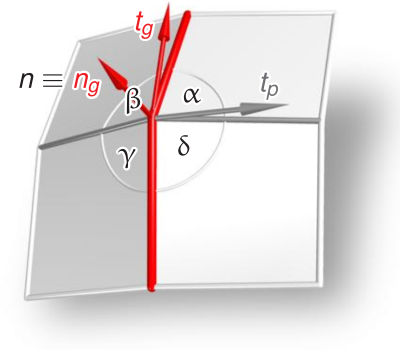
$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$

$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$

$= \langle e_+ + e_-, e_+ - e_- \rangle$

$= \langle e_+, e_+ \rangle - \langle e_+, e_- \rangle + \langle e_-, e_+ \rangle - \langle e_-, e_- \rangle$

Discrete geodesic parallel coordinates



Geodesic:

surface normal: $n = t_p \times t_g$

$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$

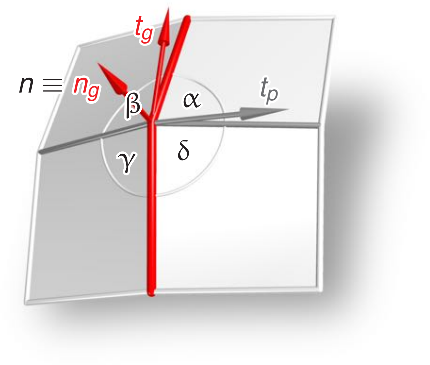
$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$

$= \langle e_+ + e_-, e_+ - e_- \rangle$

$= \langle e_+, e_+ \rangle - \langle e_+, e_- \rangle + \langle e_-, e_+ \rangle - \langle e_-, e_- \rangle$

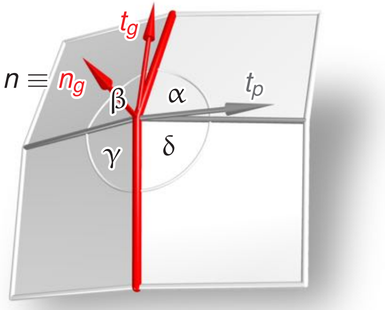
$= \cos\alpha - \cos\beta + \cos\delta - \cos\gamma$

Discrete geodesic parallel coordinates



Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

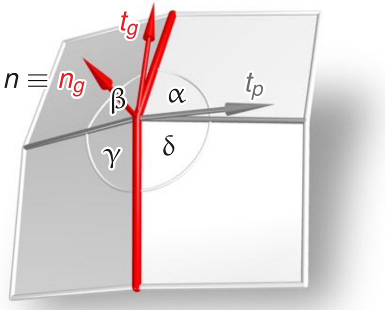
Discrete geodesic parallel coordinates



Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

Parallel:

Discrete geodesic parallel coordinates

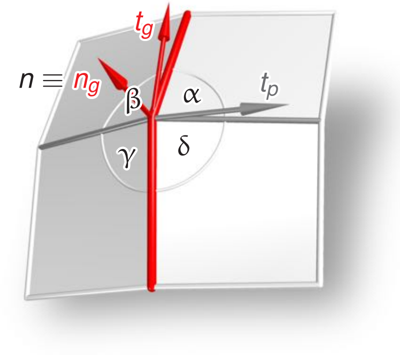


Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

Parallel:

$t_g \perp t_p \Leftrightarrow \langle t_g, t_p \rangle = 0$

Discrete geodesic parallel coordinates



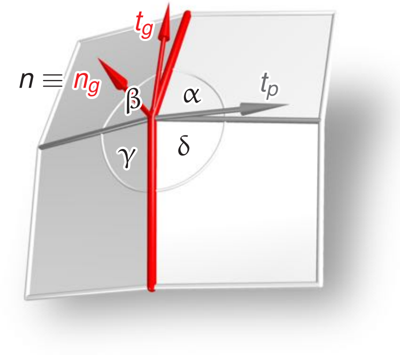
Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

Parallel:

$t_g \perp t_p \Leftrightarrow \langle t_g, t_p \rangle = 0$

$= \langle e_+ - e_-, e_+ - e_- \rangle$

Discrete geodesic parallel coordinates



Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

Parallel:

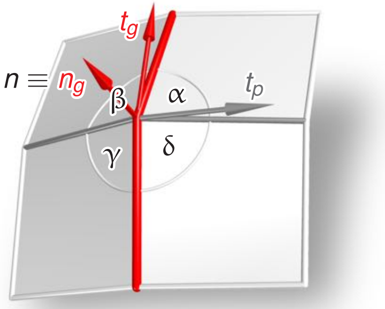
$$t_g \perp t_p \Leftrightarrow \langle t_g, t_p \rangle = 0$$

$$= \langle e_+ - e_-, e_+ - e_- \rangle$$

$$= \langle e_+, e_+ \rangle - \langle e_+, e_- \rangle + \langle e_-, e_+ \rangle - \langle e_-, e_- \rangle$$

$$= \cos\alpha - \cos\beta - \cos\gamma + \cos\delta$$

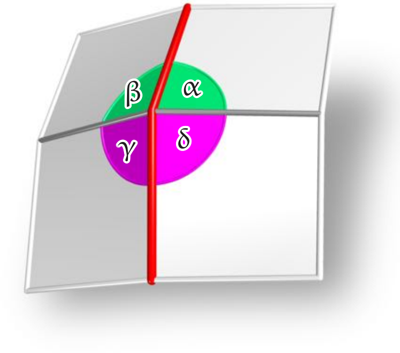
Discrete geodesic parallel coordinates



Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

Parallel: $\cos\alpha + \cos\gamma = \cos\beta + \cos\delta$

Discrete geodesic parallel coordinates



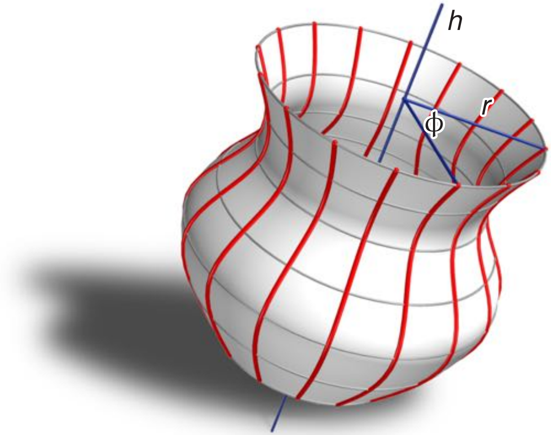
Geodesic: $\cos\alpha + \cos\delta = \cos\beta + \cos\gamma$

Parallel: $\cos\alpha + \cos\gamma = \cos\beta + \cos\delta$

Geodesic parallel:

$$\alpha = \beta, \quad \gamma = \delta$$

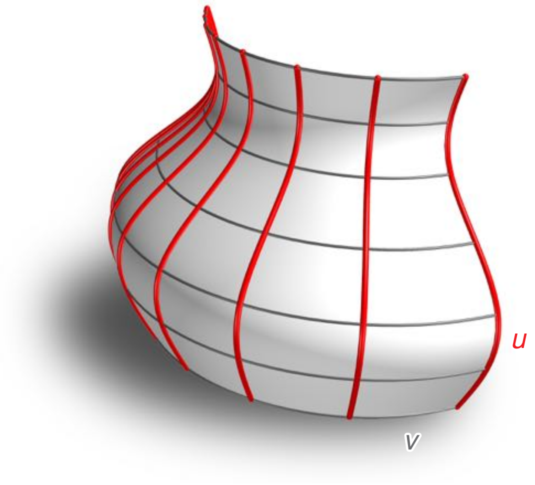
Rotational surfaces



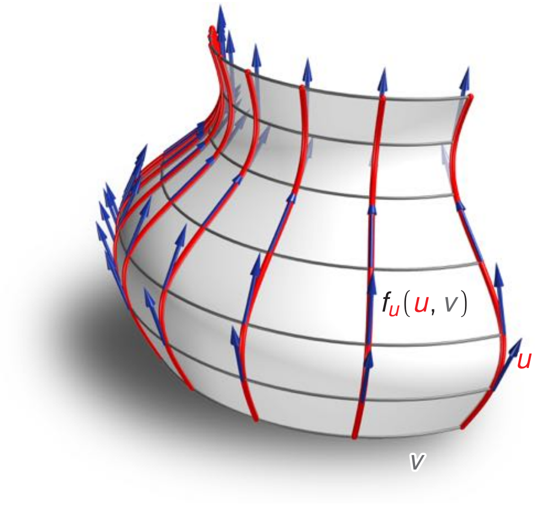
Geodesic parallel parametrization:

$$f = \big(r(u)\cos\phi(v),\, r(u)\sin\phi(v),\, h(u) \big)$$

Rotational isometric surfaces

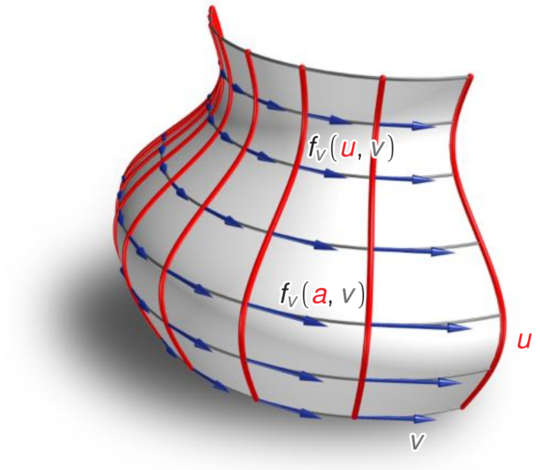


Rotational isometric surfaces



$$\partial_v \| f_u(u, v) \| = 0$$

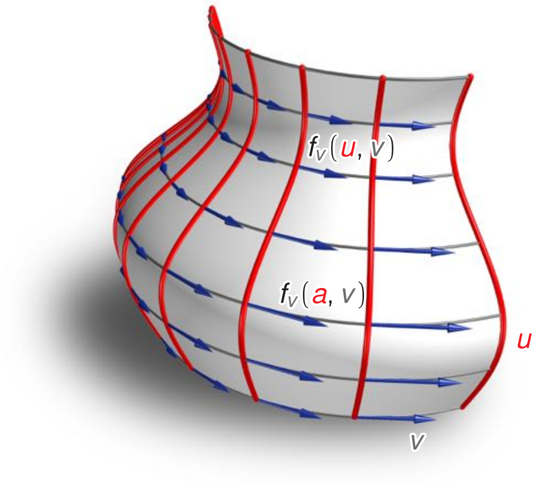
Rotational isometric surfaces



$$\partial_v \|f_u(u, v)\| = 0$$

$$r(u) = \frac{\|f_v(u, v)\|}{\|f_v(a, v)\|}$$

Rotational isometric surfaces



$$\partial_v \|f_u(u, v)\| = 0$$

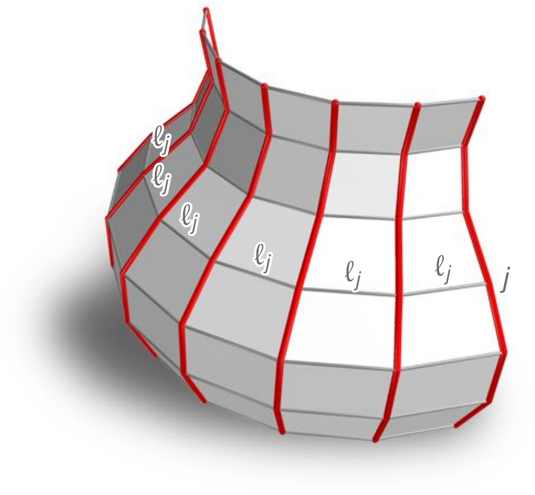
$$r(u) = \frac{\|f_v(u, v)\|}{\|f_v(a, v)\|}$$

$$\Downarrow$$

$$\hat{f} = \Big(r(u)\cos\phi(v), r(u)\sin\phi(v), h(u)\Big)$$

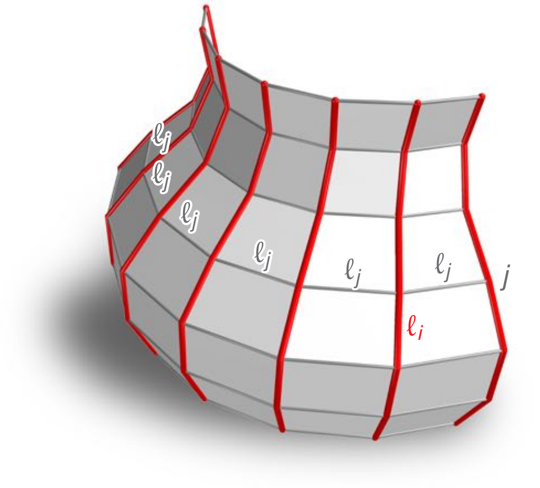
$$h(u) = \int_{u_0}^{u} \sqrt{\|f_u(t, v)\| + r'(t)}\, \mathrm{d}t$$

Discrete rotational isometric surfaces



Equal edge length along parallel polylines

Discrete rotational isometric surfaces



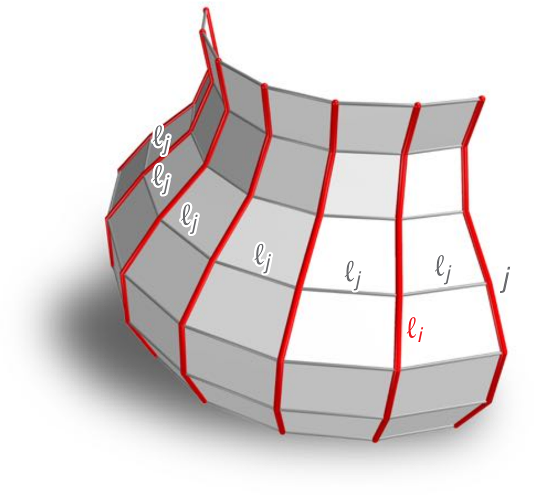
Equal edge length along parallel polylines

$$\Downarrow$$

Isometric rotational surface:

$$r_j = \frac{\ell_j}{2\sin(\pi/n_j)}$$

Discrete rotational isometric surfaces



Equal edge length along parallel polylines

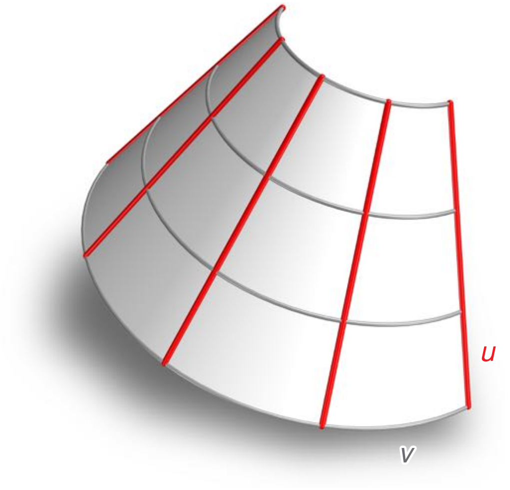
$$\Downarrow$$

Isometric rotational surface:

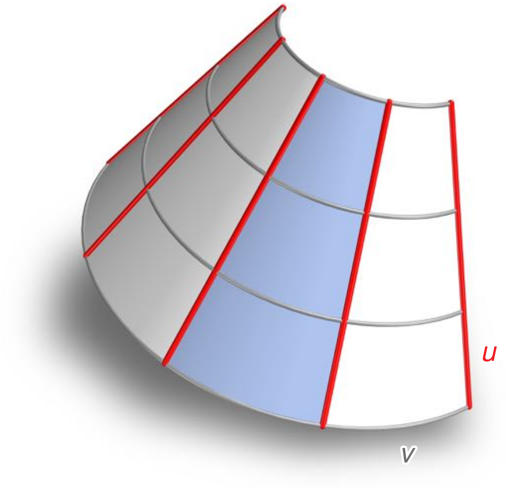
$$r_j = \frac{\ell_j}{2\sin(\pi/n_j)}$$

$$\Delta h_j = \sqrt{\ell_i{}^2 - (r_{j-1} - r_j)^2}$$

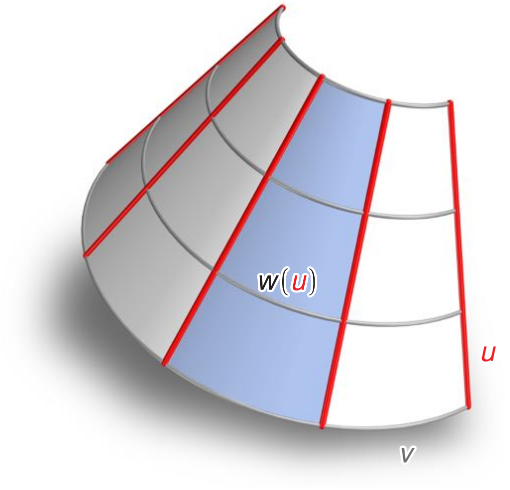
Developable surfaces



Developable surfaces



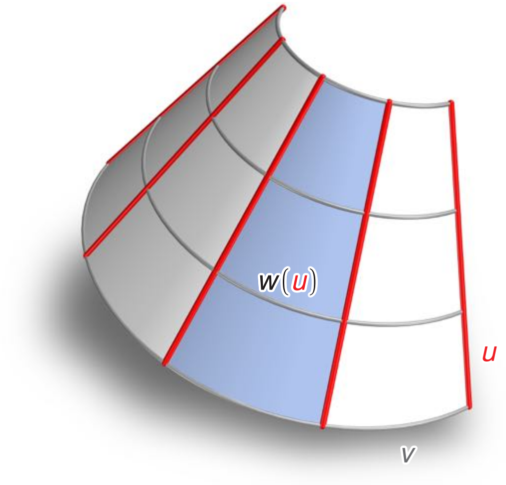
Developable surfaces



Strip width:

$$w(u) = \int_{v}^{v+\epsilon} \|f_v(u,t)\| \, \mathrm{d}t$$

Developable surfaces



Strip width:

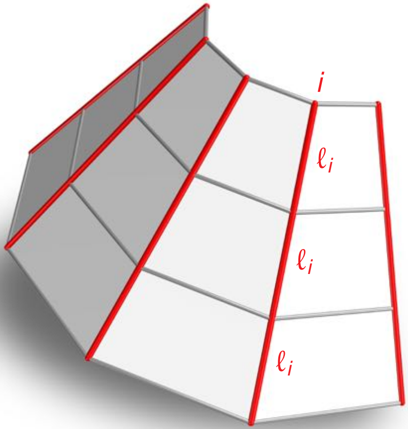
$$w(u) = \int_v^{v+\epsilon} \|f_v(u,t)\| \, \mathrm{d}t$$

Jacobi equation:

(constant speed parametrization along geodesics)

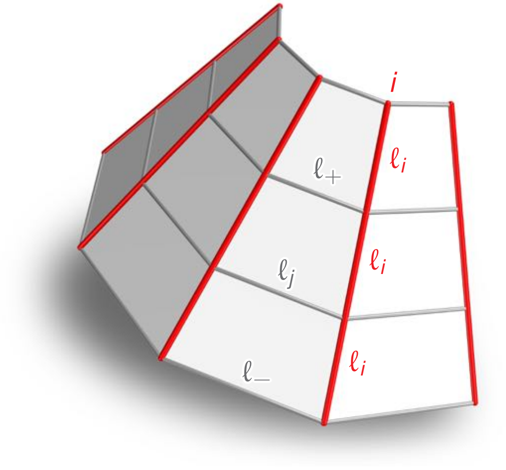
$$K = 0 \Rightarrow \partial_{uu} w(u) = 0$$

Discrete developable surfaces



Equal edge length along geodesic polylines

Discrete developable surfaces



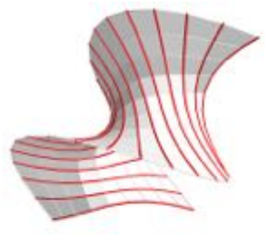
Equal edge length along geodesic polylines

$$\Downarrow$$

Discrete Jacobi equation:

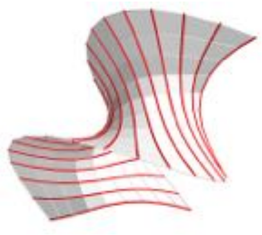
$$\ell_j = \frac{\ell_- + \ell_+}{2}$$

Modeling with geodesic parallel meshes



- Geodesic parallel angles

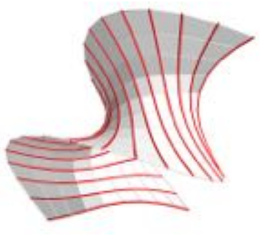
Modeling with geodesic parallel meshes



- Geodesic parallel angles

- Geodesic parallel angles
- Equal edge length along parallel polylines

Modeling with geodesic parallel meshes



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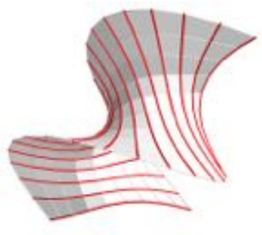
Modeling with geodesic parallel meshes



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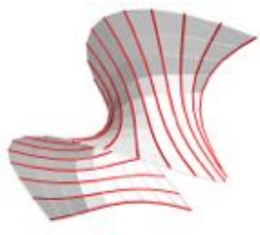
Guided projection [Tang et al. 2014]

Geodesic parallel meshes



- Geodesic parallel angles

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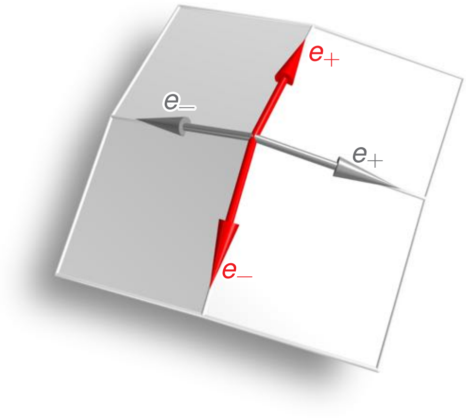
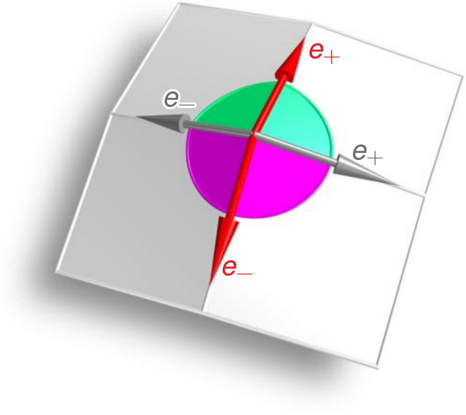
Geodesic parallel meshes

Constraints



Geodesic parallel meshes

Constraints



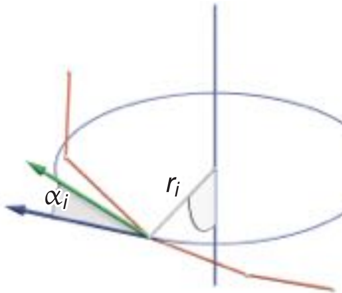
Geodesic parallel angles:

$$\langle e_+, e_- \rangle - \langle e_+, e_+ \rangle = 0,$$

$$\langle e_-, e_- \rangle - \langle e_-, e_+ \rangle = 0.$$

Geodesic parallel meshes

Verification

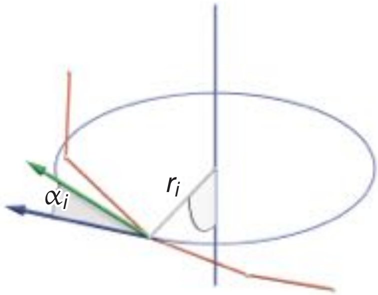


Clairaut's relation:

$$r_i \cos \alpha_i = \text{const}$$

Geodesic parallel meshes

Verification



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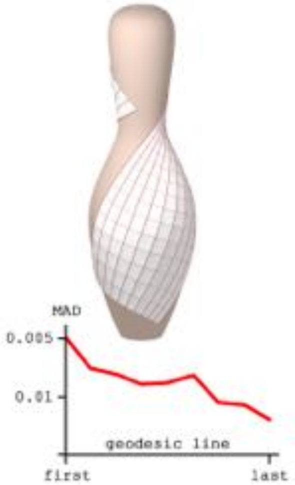

Geodesic parallel meshes

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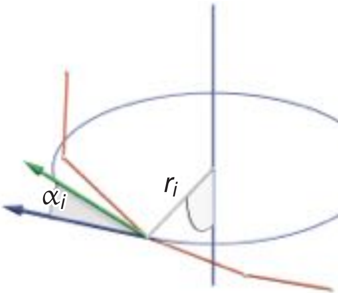

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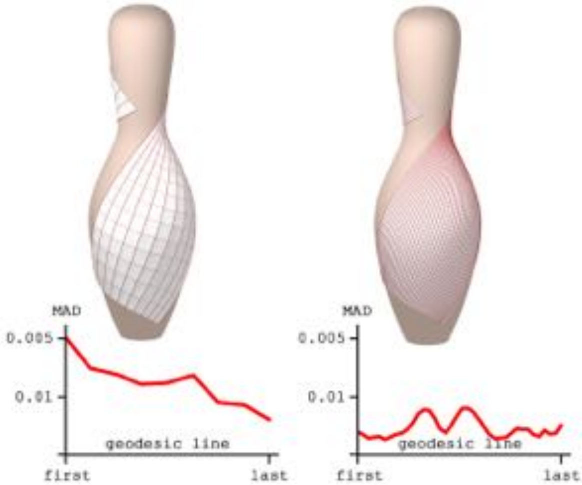
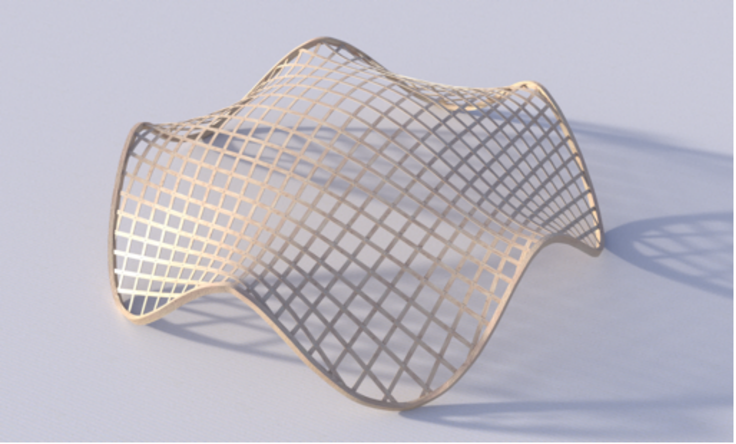


Geodesic parallel meshes

Geodesic gridshells



Geodesic parallel meshes

Geodesic gridshells

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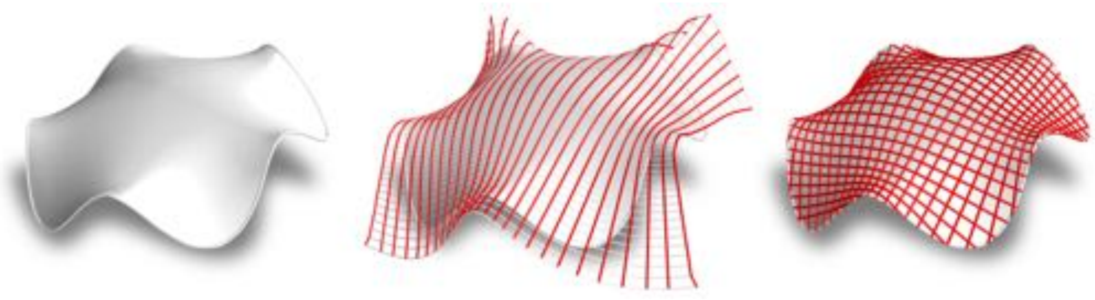
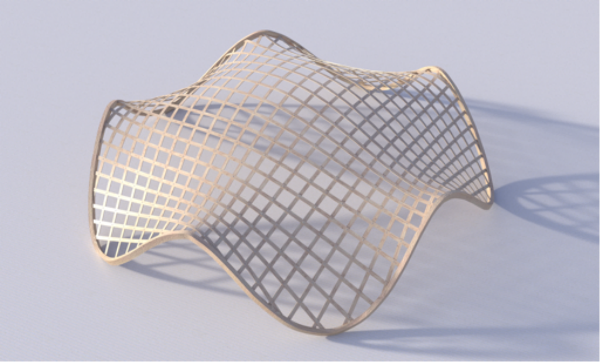


Geodesic parallel meshes

Geodesic gridshells



Rotational isometric meshes

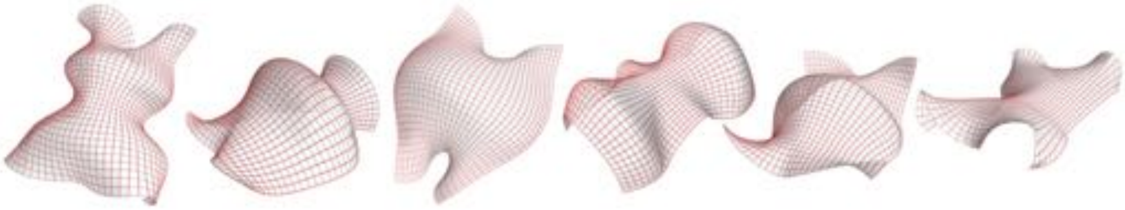
- Geodesic parallel angles

- Geodesic parallel angles
- Equal edge length along parallel polylines

- Geodesic parallel angles
- Equal edge length along geodesic polylines
- Discrete Jacobi equation

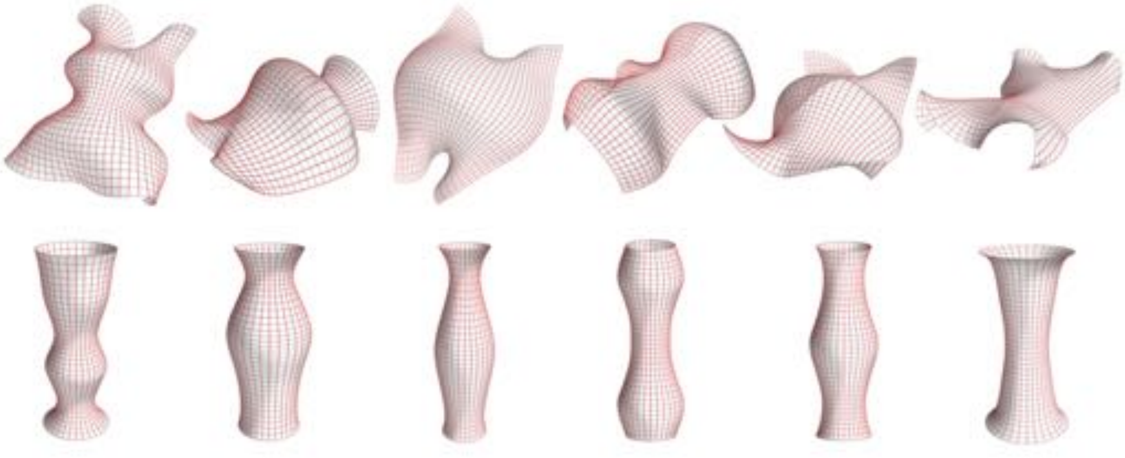
Rotational isometric meshes

Modeling



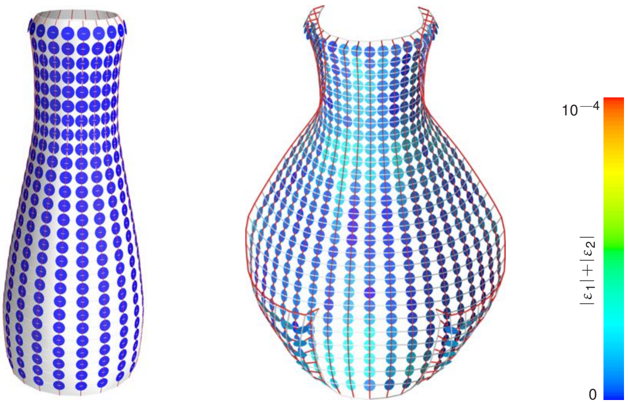
Rotational isometric meshes

Modeling



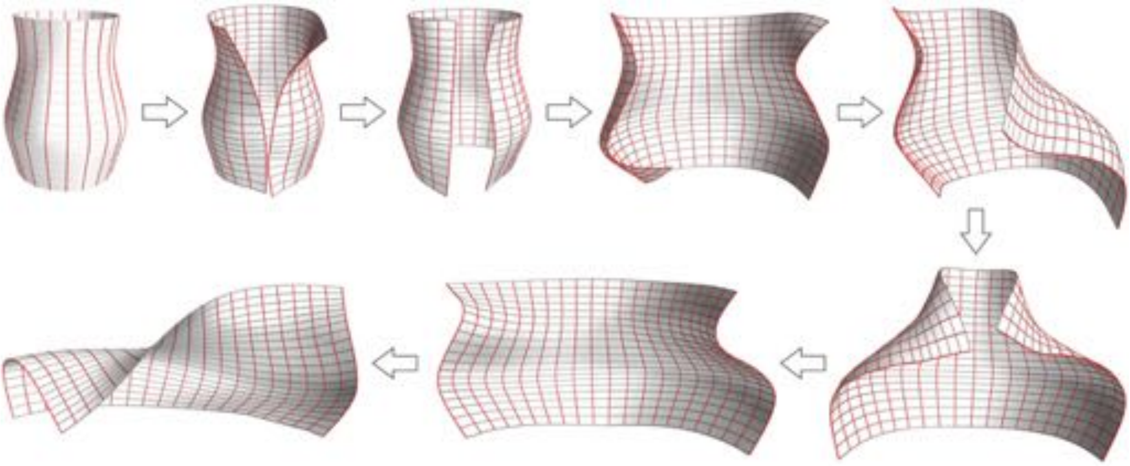
Rotational isometric meshes

Isometry error estimation



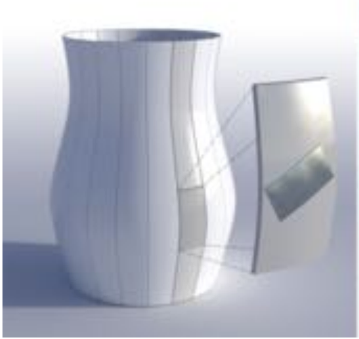
Rotational isometric meshes

Isometric deformation



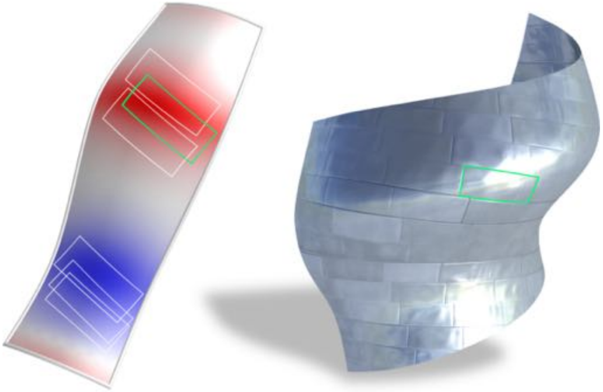
Rotational isometric meshes

Surfaces of revolution as molds



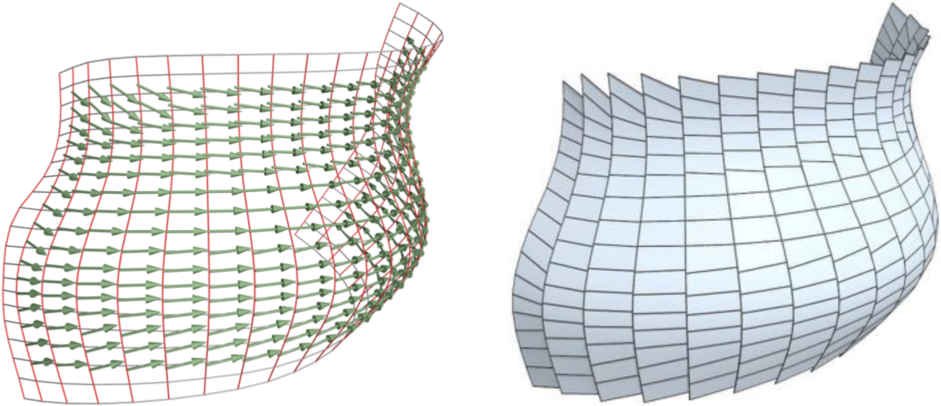
Rotational isometric meshes

Surfaces of revolution as molds



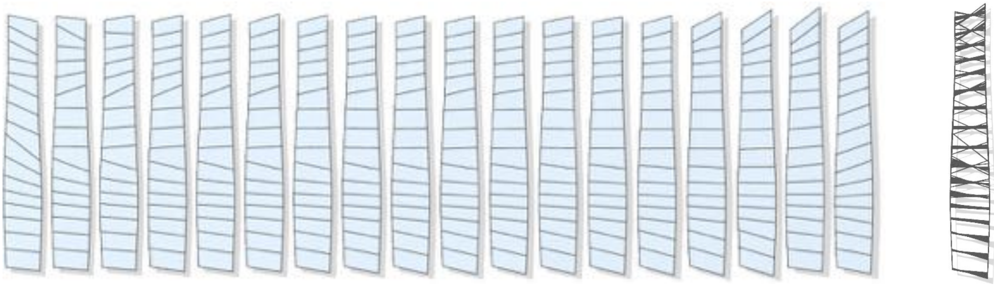
Rotational isometric meshes

Repetitive strip models



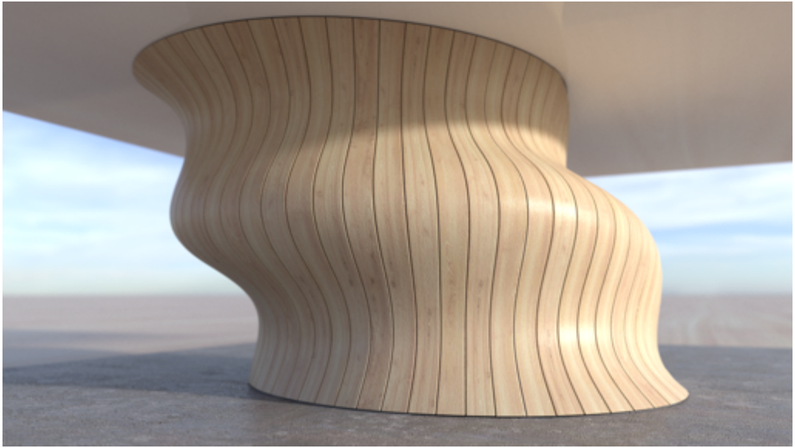
Rotational isometric meshes

Repetitive strip models



Rotational isometric meshes

Repetitive strip models



Nearly developable meshes

- Geodesic parallel angles

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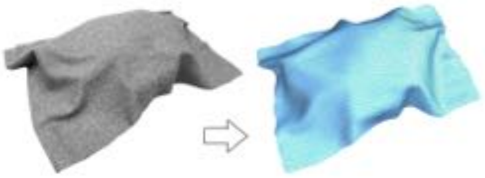
Nearly developable meshes

Approximation

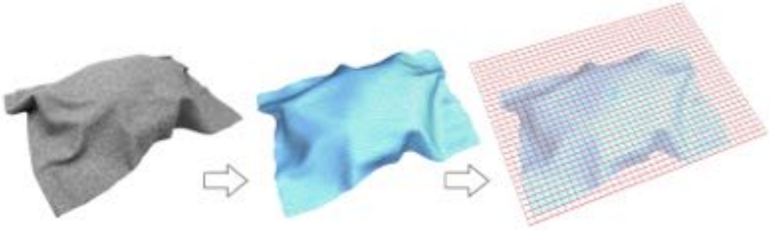
Nearly developable meshes

Approximation



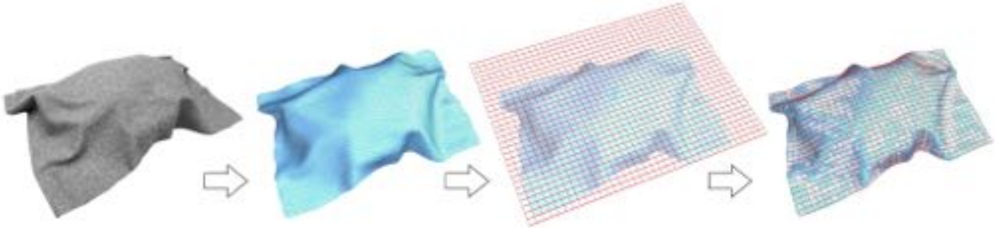
Nearly developable meshes

Approximation



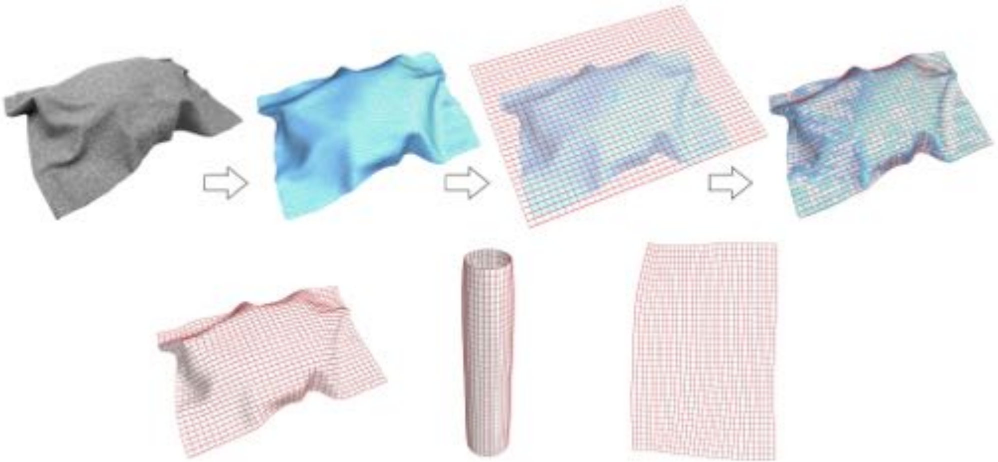
Nearly developable meshes

Approximation



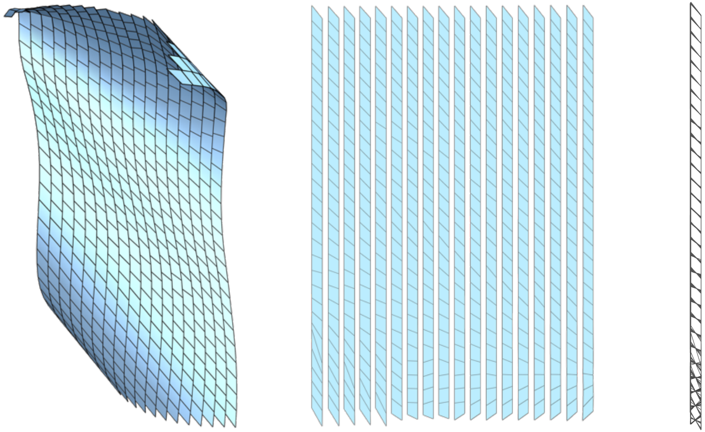
Nearly developable meshes

Approximation



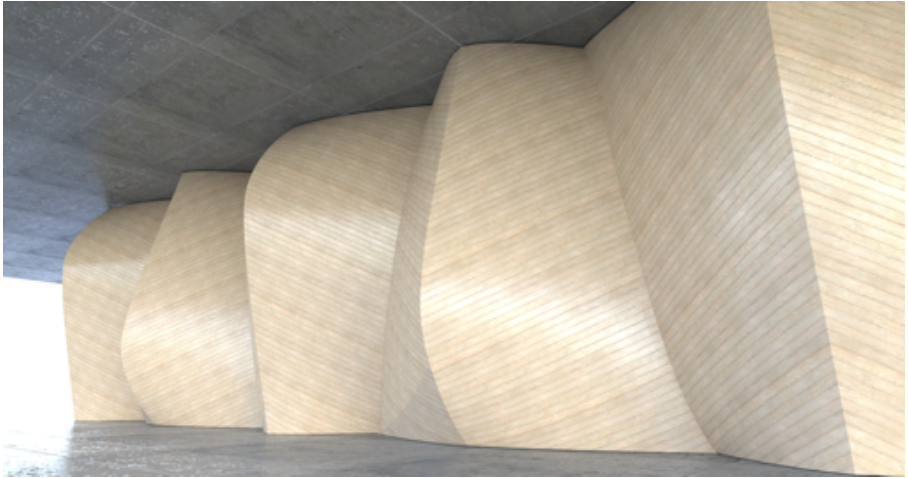
Nearly developable meshes

Verification



Nearly developable meshes

Cladding



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