



## 1 Introduction

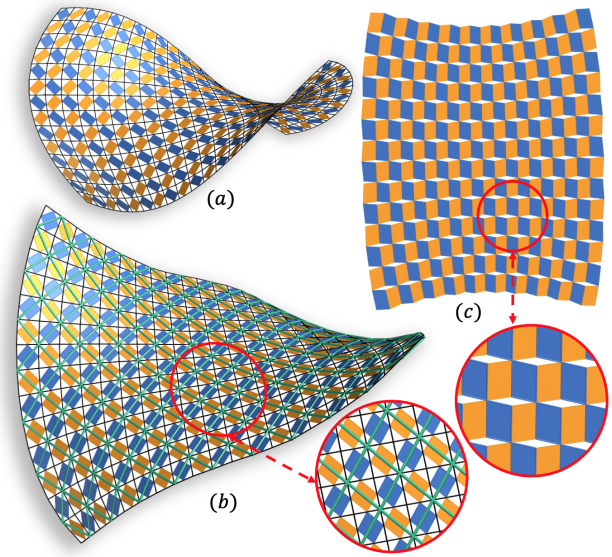
Auxetic metamaterials—whose negative Poisson’s ratio enables unique mechanical responses—have attracted intense interest for biomedical, aerospace and soft robotics. Existing strategies (rotating rigid units, star lattices, re-entrant honeycombs, chiral frameworks, and topology-optimized lattices) are hindered by hinge stress, anisotropy, scale-up difficulties, high computational cost and fabrication complexity [Zhang et al. 2025]. Most are case-specific and lack a systematic route to large-scale, low-cost production.

Recent multi-state shape-morphing auxetics [Jiang et al. 2022] show that geometric programming based on conformal geometry can unify inverse design and continuous transition between extreme auxetic states, breaking the bottleneck of traditional bottom-up searches [Dudte et al. 2023]. Building on this insight, we propose a controllable kirigami framework governed by explicit geometric rules. Every rigid auxetic quad panel is identical in shape, cutting tooling costs and enabling reliable mass production, while the parametric design space delivers predictable, programmable transitions between fully open and fully closed auxetic extremes.

## 2 Method

Guaranteeing panel regularity while preserving fabrication-friendly simplicity remains non-trivial; we therefore seek a programmable kirigami framework that delivers auxetic behavior across multi-state configurations. We start from a single spatial quad that contains an inscribed parallelogram whose edges slide with a constant ratio  $t$  or  $\bar{t} = 1 - t \in [0, 1]$ , and extend this rule to a quad net by marking every second face (Fig. 1). The resulting *Checkerboard Pattern* (CBP) [Jiang et al. 2019] furnishes the initial kirigami layout (Fig. 2-(a)). To make all panels mutually similar, we enforce global geometric constraints: every inscribed parallelogram shares the same interior angle and its diagonals intersect at a uniform angle—equivalently, fixed shape ratios  $(\lambda, \mu)$  tied to an *isogonal net* [Wang et al. 2025]. We then determine when the framework stays open (Fig. 1-(d)) and how it can close along two distinct directions, yielding two different closed states (Fig. 1-(c,e)). The following provides the computational design foundations:

- Rigid auxetic quad panels are inscribed parallelograms of the control net, split into two classes:  $t$ -parallelograms (Fig. 1-(a)) and  $\bar{t}$ -parallelograms (Fig. 1-(b)) distinguished by the cyclic edge-gliding ratios  $(t, 1 - t, t, 1 - t)$  or  $(1 - t, t, 1 - t, t)$  that position their vertices along successive edges. At  $t = 0.5$  the CBP coincides with a mid-edge subdivision [Jiang et al. 2019].
- Fixing the shape ratios  $(\lambda, \mu)$  for every  $(t, \bar{t})$ -parallelogram makes all auxetic quads mutually similar and generates an isogonal diagonal net; the single triple  $(t, \lambda, \mu)$  thereby governs the scale and shape of every auxetic quad globally.
- Solve for the global variable set  $\mathcal{X} = \{\mathbf{V}, \mathbf{P}, t, \lambda, \mu\}$  under geometric constraints that encode (i) prescribed edge-gliding ratios, (ii) mutual similarity of all parallelograms, (iii) shrinkable checkerboard holes, and (iv) any user-specified objectives such as shape approximation;  $\mathbf{V}$  are control-mesh vertices,  $\mathbf{P}$  are the edge-gliding vertices of all parallelograms, and  $(t, \lambda, \mu)$  are three scalar parameters. Fully open and fully



**Figure 2:** (a) A CBP whose alternating facets (blue and yellow) are  $(0.4, 0.6)$ -parallelograms is extracted from a general quad net (black). (b) The parallelograms are optimized to be similar while the checkerboard holes (white) are tuned to become collapsible seams; after unrolling they shrink to line segments as shown in (c). The diagonal net (green) of the black control net in (b) is isogonal: its two diagonal families meet at a constant angle. (c) Planar kirigami pattern corresponding to (b); all matching parallelograms are isometric.

closed auxetic extremes are obtained by enforcing isometric correspondence between states (Fig. 2).

## 3 Results and future work

We present a controllable kirigami framework in which every rigid quad panel—an inscribed parallelogram of a prescribed-ratio quad net—is mutually similar, slashing digital-fabrication costs. The framework exploits the isogonal diagonal net of the underlying control net, whose geometric properties guide shape exploration.

Ongoing work will use discrete differential geometry to compute auxetic kirigami patterns from isogonal nets, enabling programmable switching between closed and open configurations ( $2D \leftrightarrow 3D$  and  $3D \leftrightarrow 3D$ ) toward a fully designable system for auxetic structures. Future work will incorporate material and force properties for applied fabrication.

## References

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