جامعة الملك عبدالله للعلوم والتقنية
King Abdullah University of Science and Technology | VCC VISUAL COMPUTING CENTER

Designing asymptotic geodesic hybrid gridshells

Curves and Surfaces 2022

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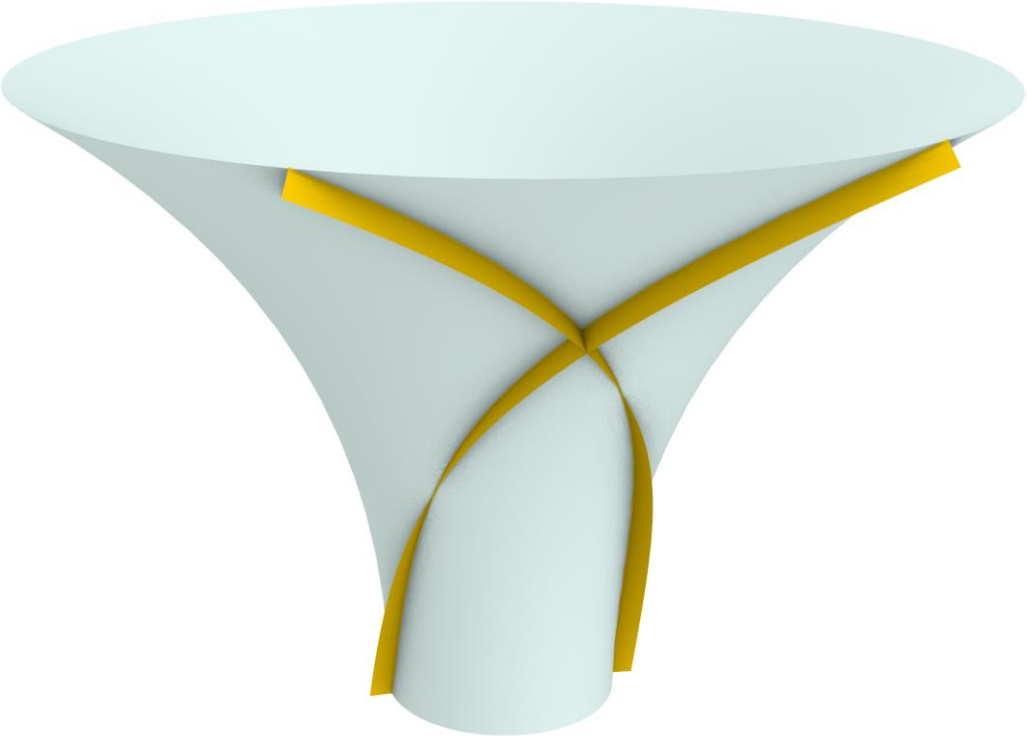
June 23, Arcachon

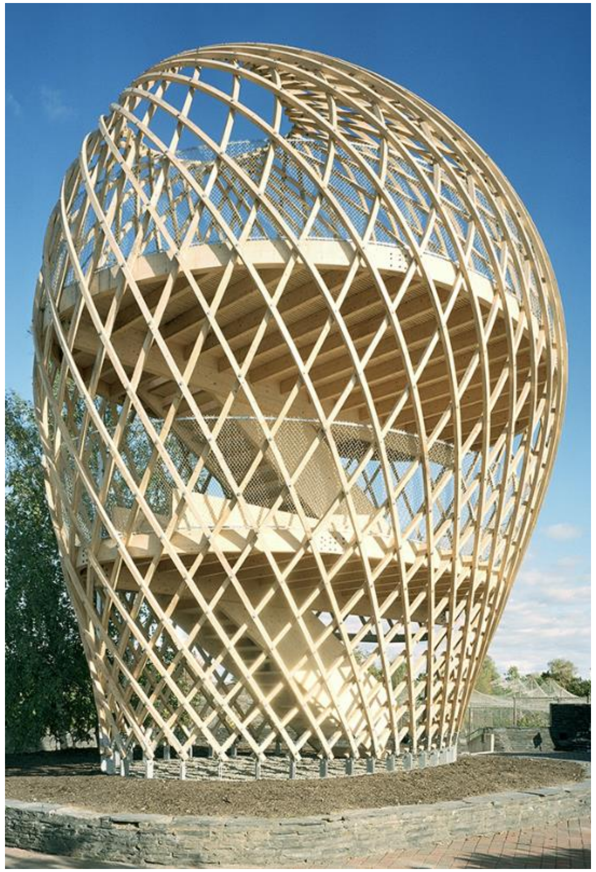
Introduction

Geodesic strips

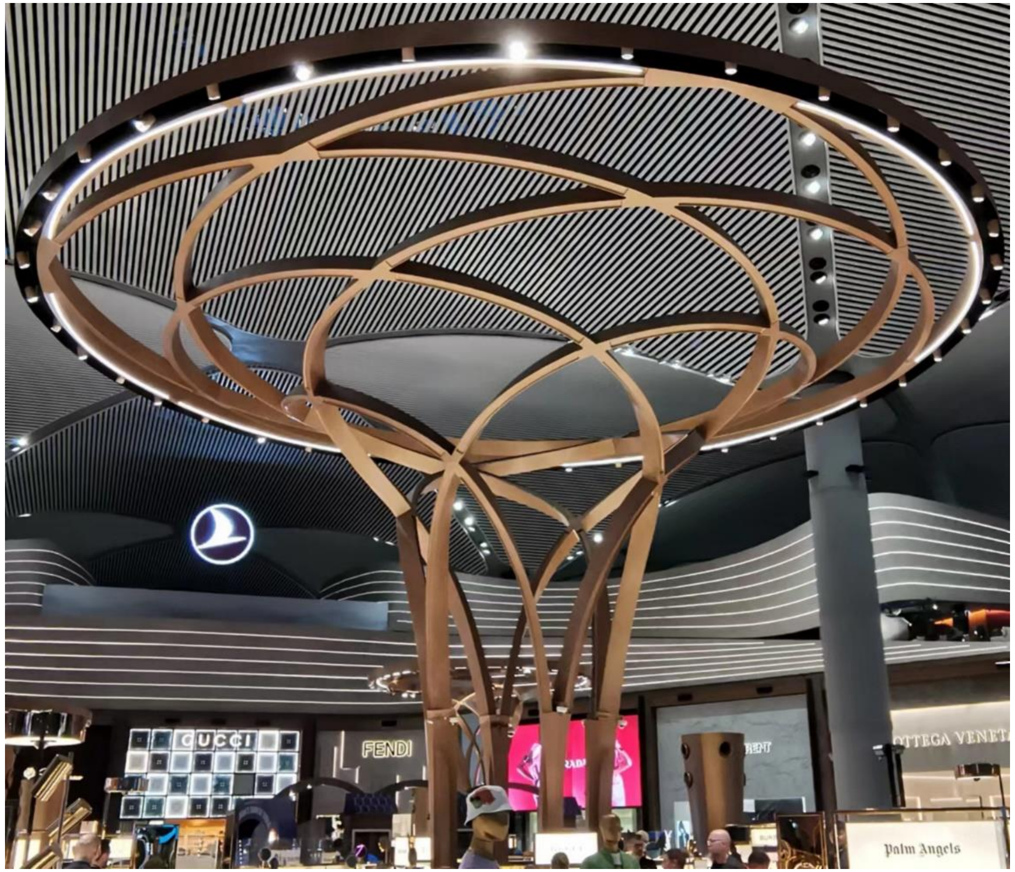


Asymptotic strips

Introduction

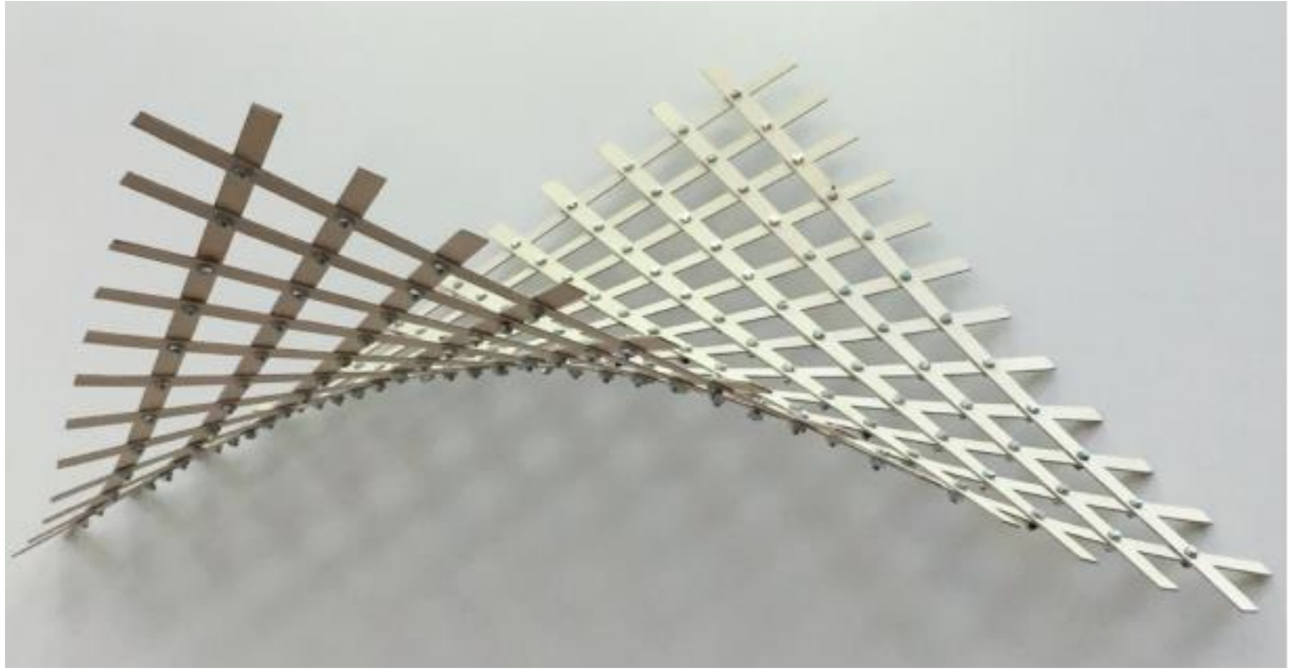


Geodesic gridshell

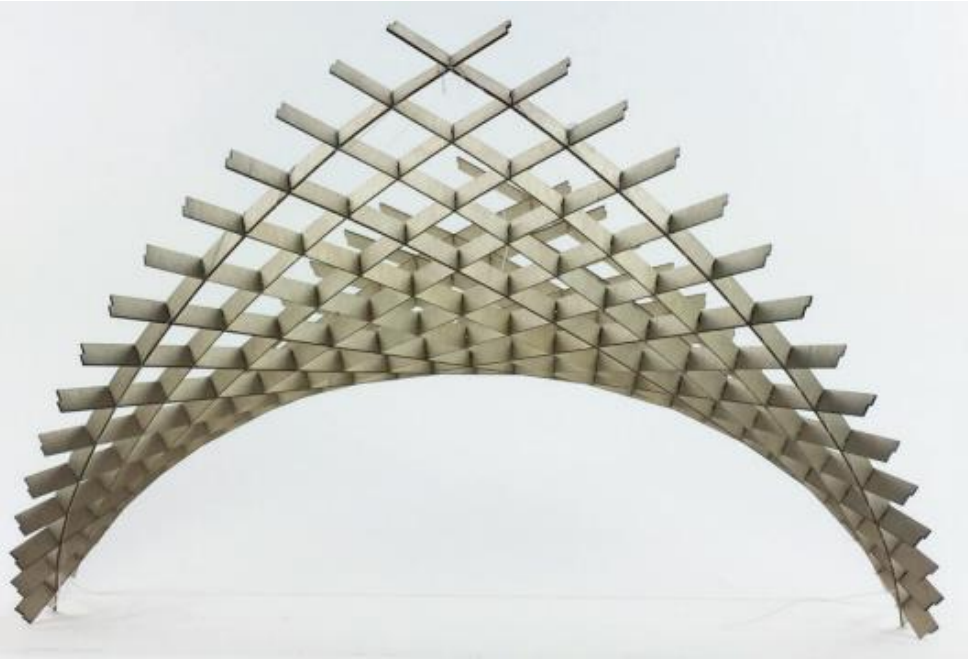


Asymptotic gridshell

Introduction



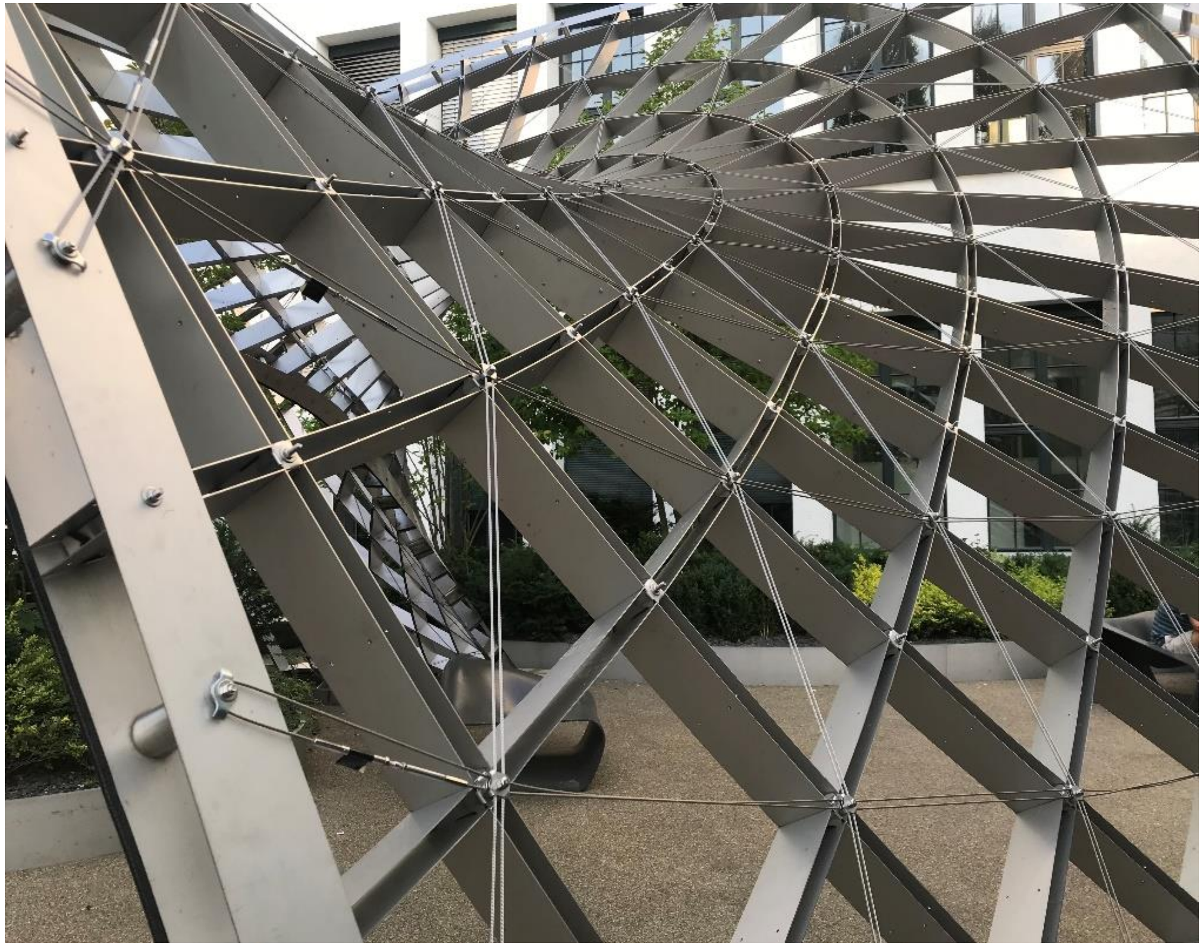
Geodesic gridshell



Asymptotic gridshell

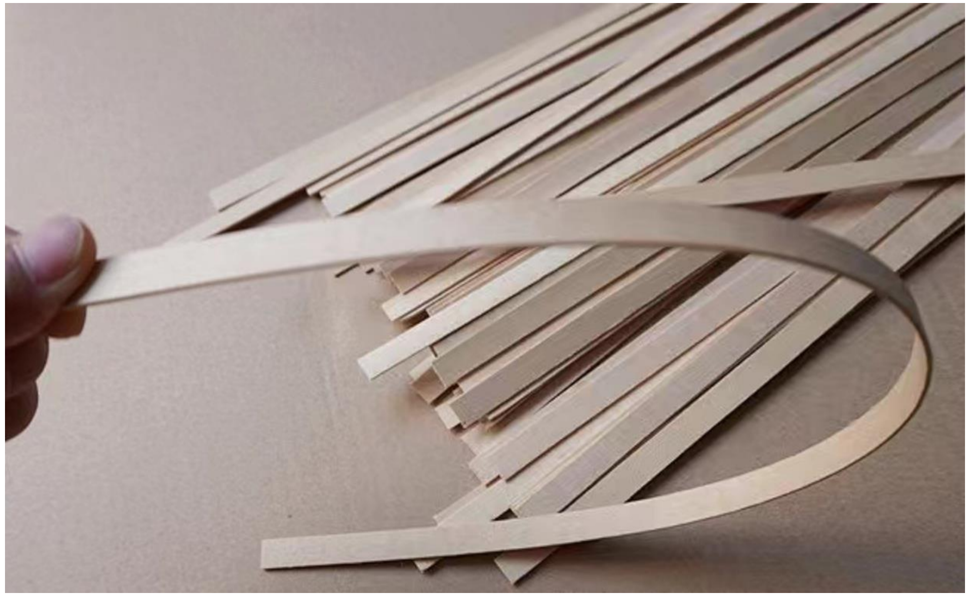
Motivation

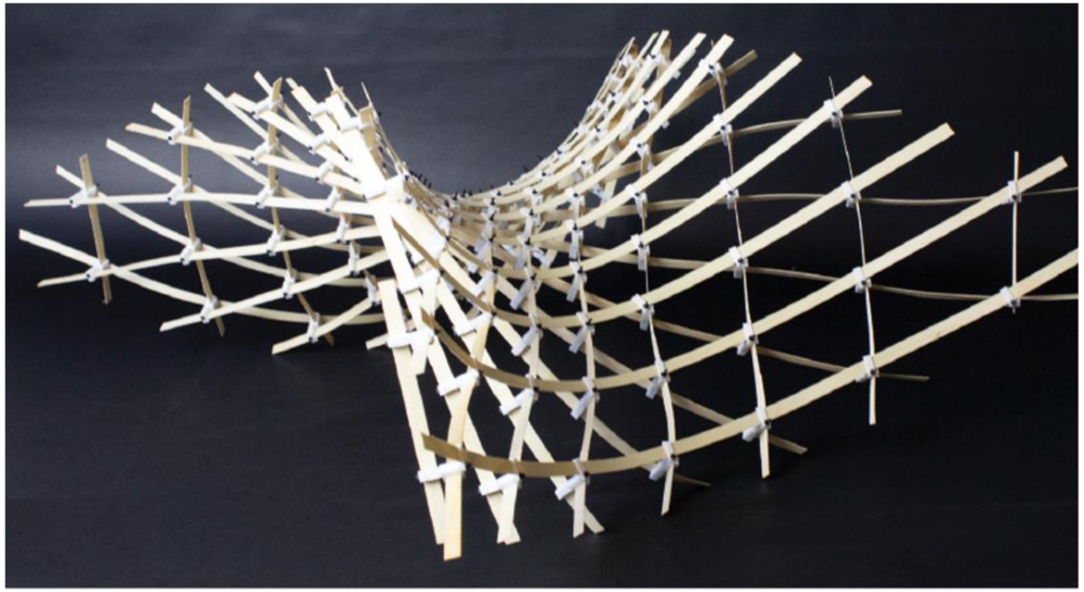
Expo Hannover Pavillion



The Inside/Out Gridshell, TUM

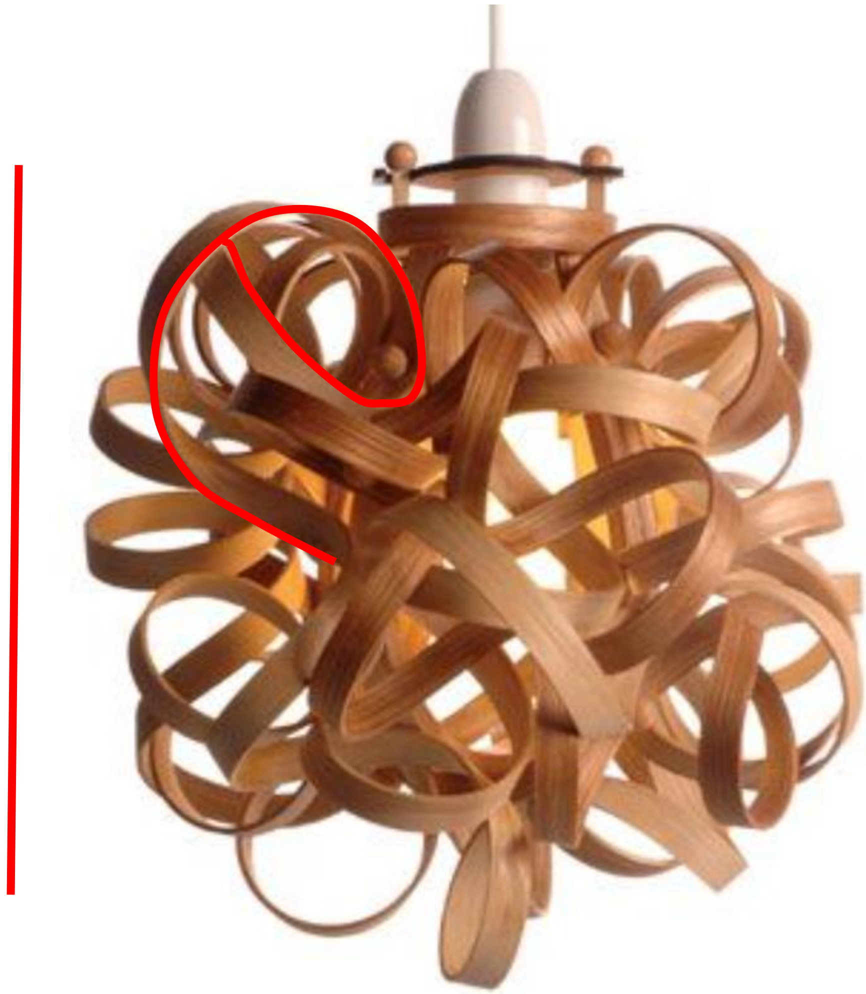
Goal

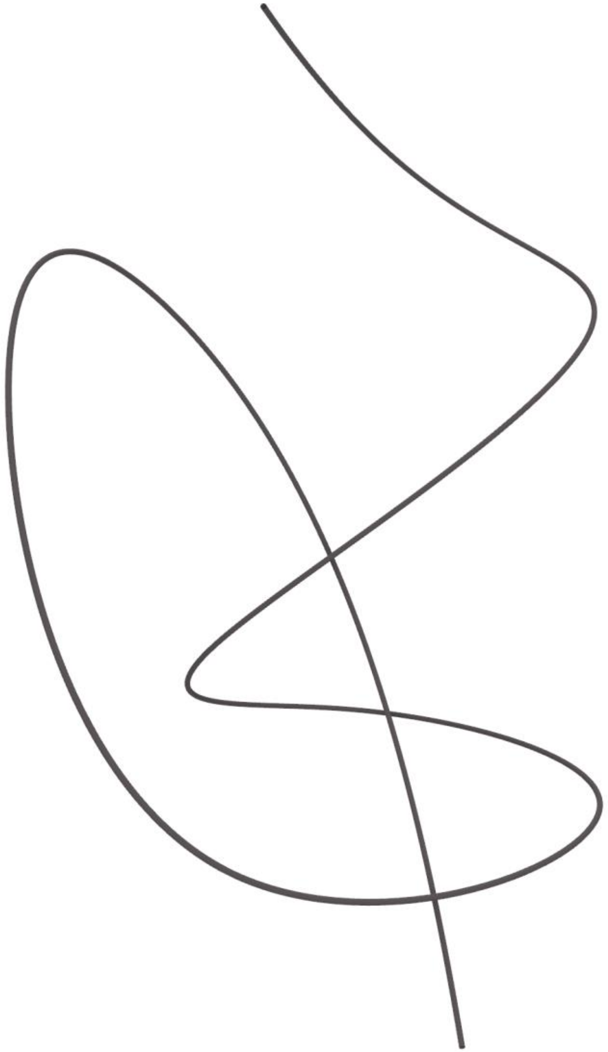




Elementary Differential Geometry

Straight development

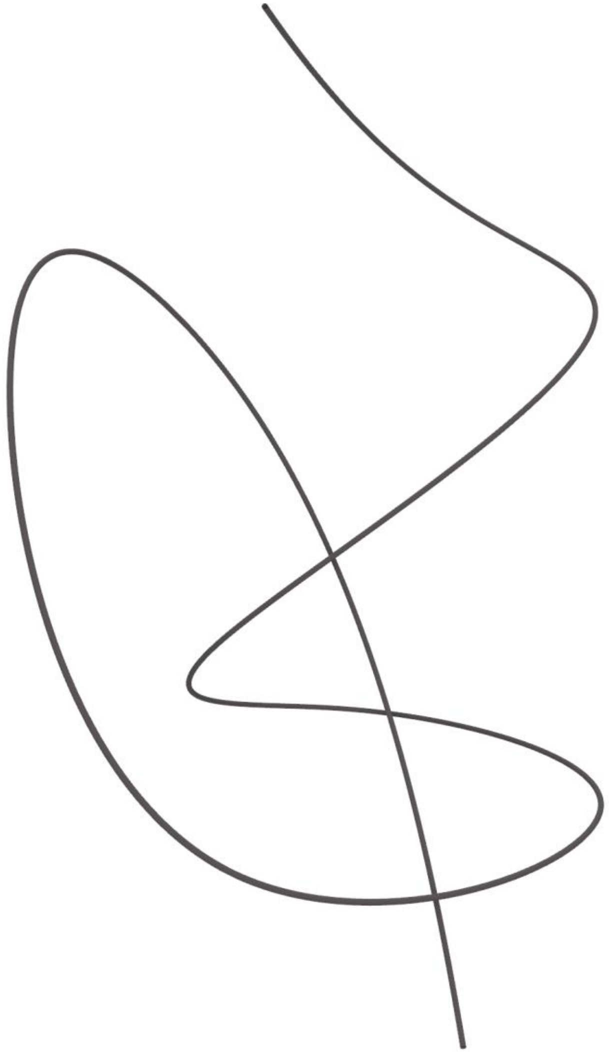


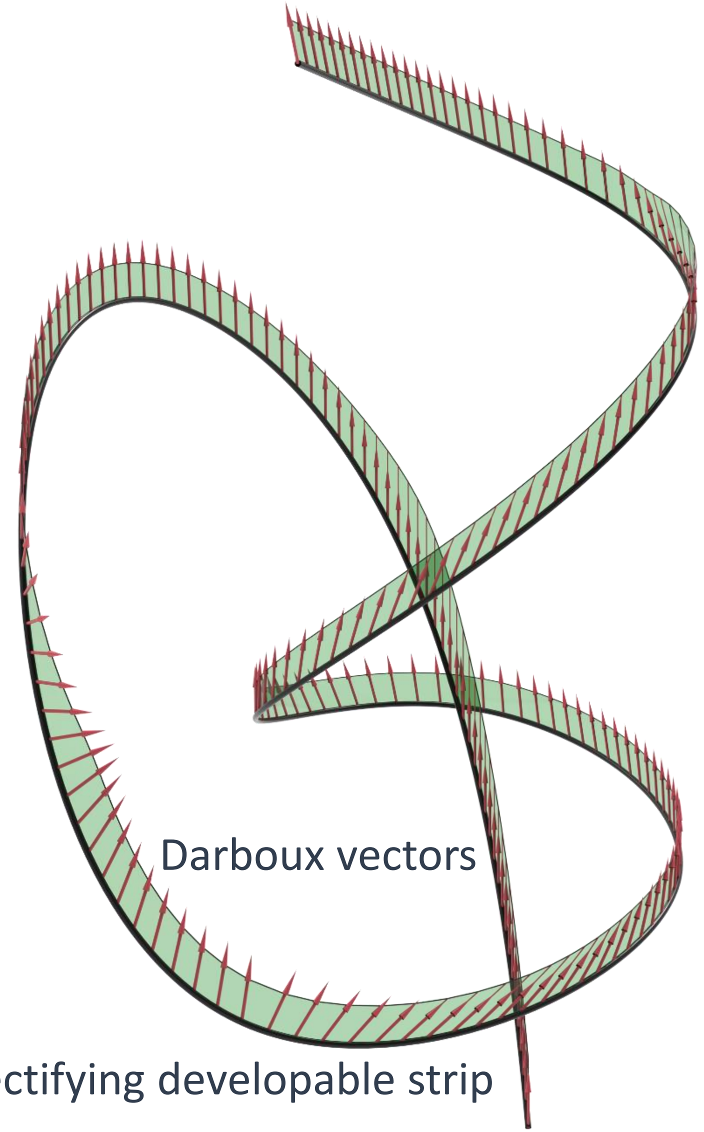


?

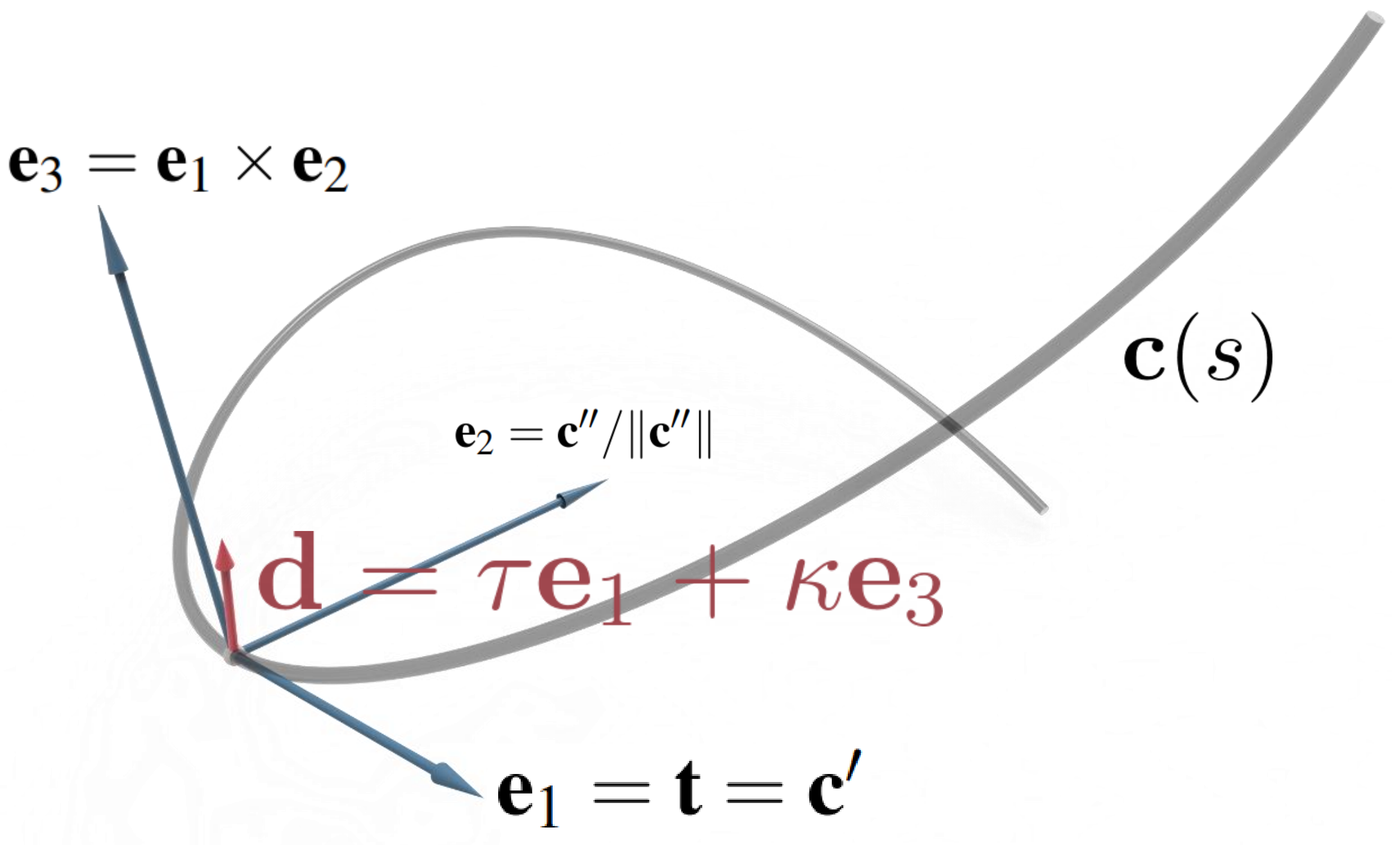
- Developable surface
- Pass through it
- Straight development

Straight development





Darboux vector



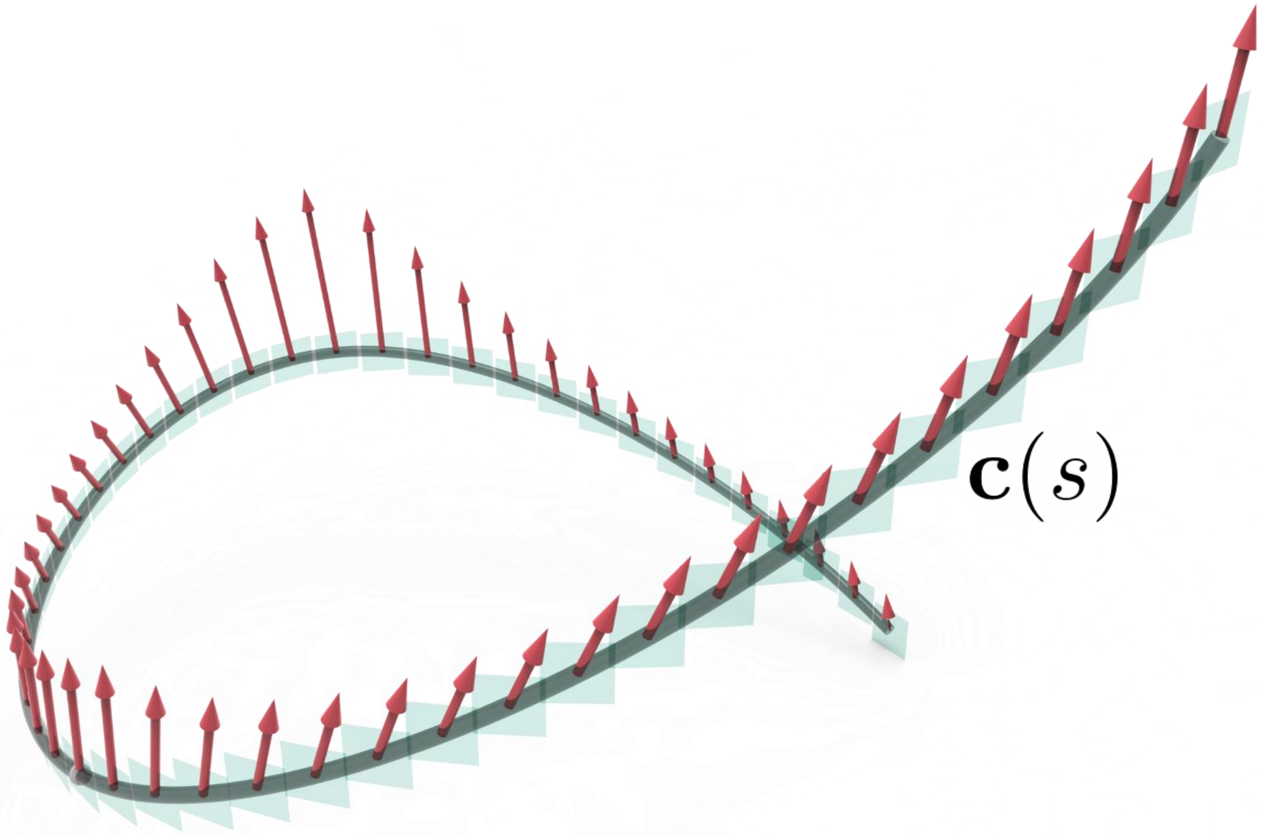
$$(\mathbf{e}_1(s), \mathbf{e}_2(s), \mathbf{e}_3(s))$$

$$\begin{bmatrix} \mathbf{e}_1' \\ \mathbf{e}_2' \\ \mathbf{e}_3' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix}$$

κ: curvature
τ: torsion

$$\mathbf{e}_i' = \mathbf{d} \times \mathbf{e}_i, \quad i = 1, 2, 3$$

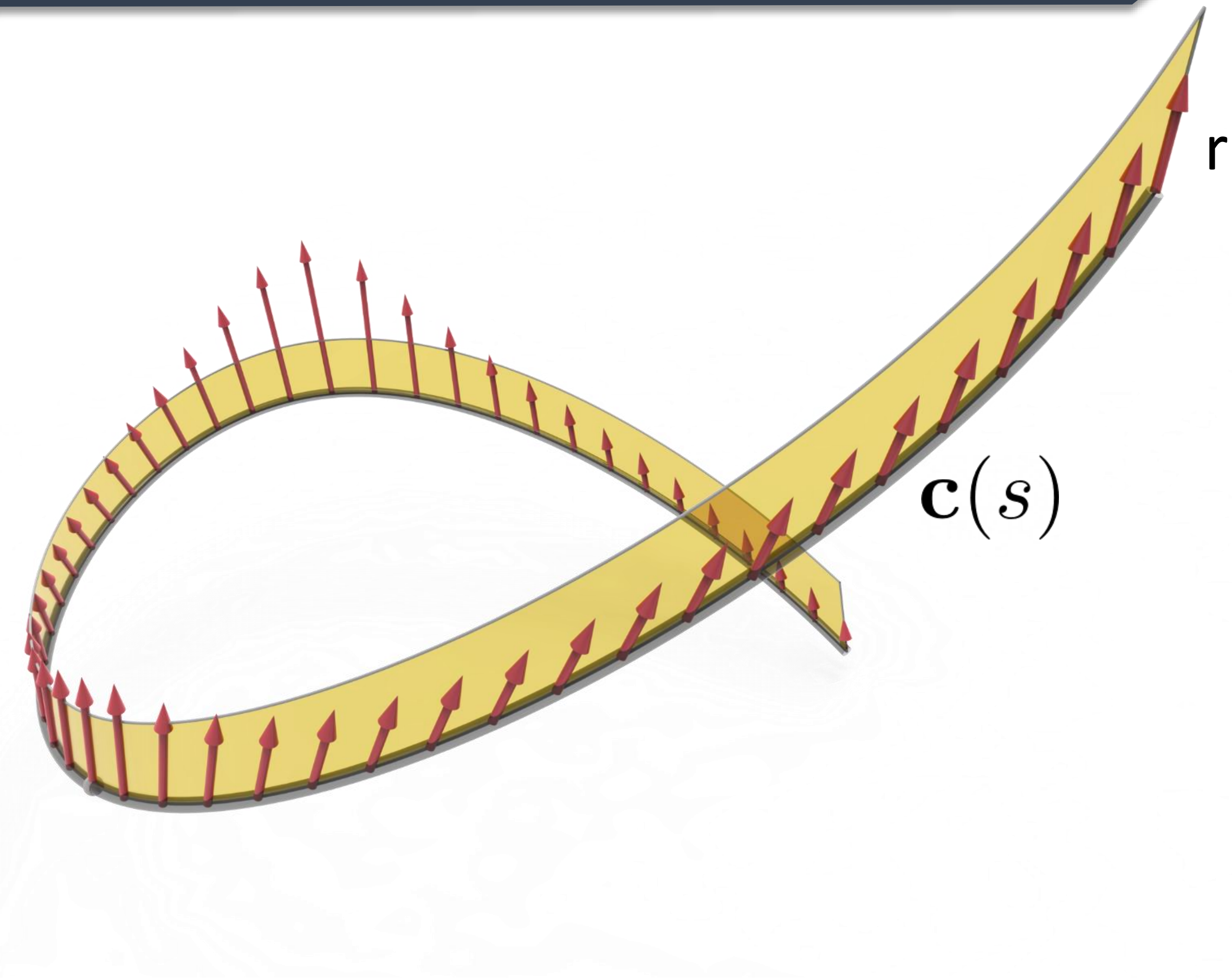
Darboux vector



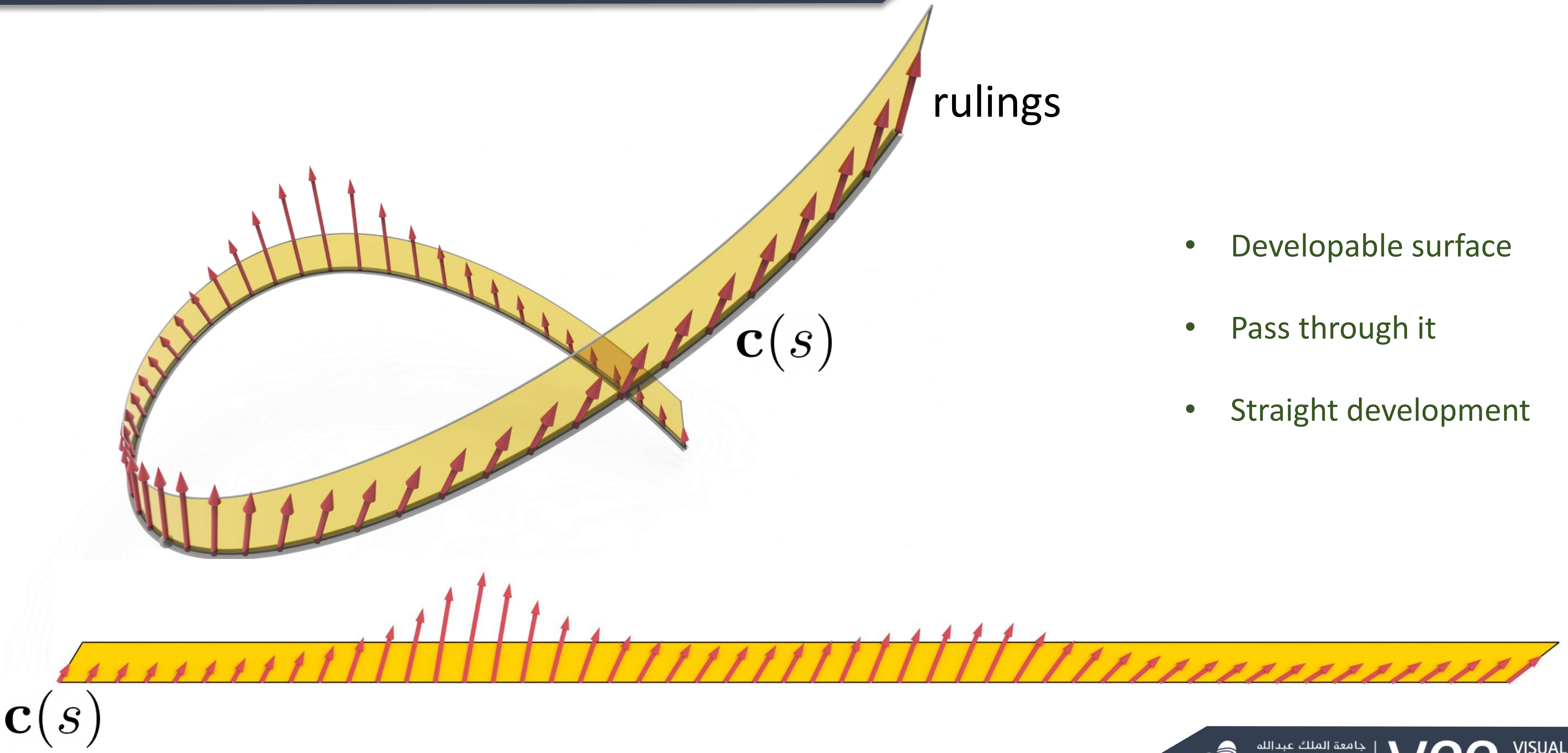
rectifying planes

$$\mathbf{d} = \tau \mathbf{e}_1 + \kappa \mathbf{e}_3$$

Rectifying developable surface

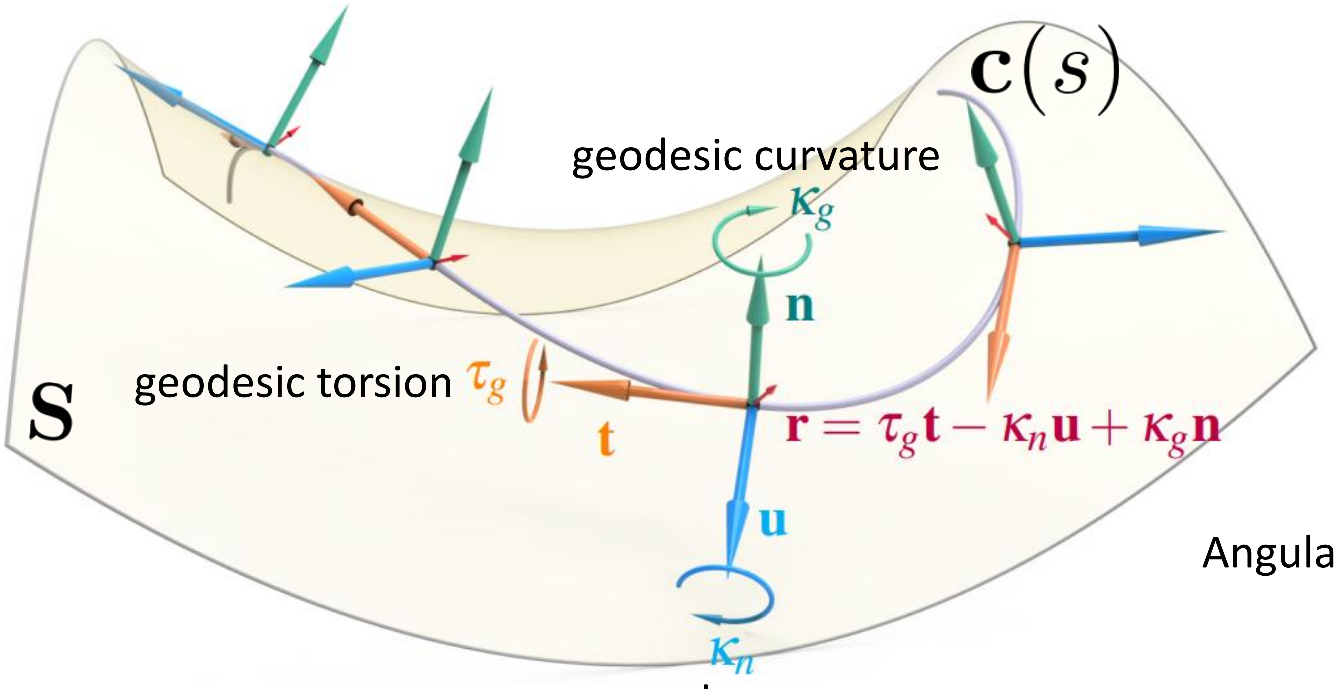


Rectifying developable surface



- Developable surface
- Pass through it
- Straight development

Darboux Frame



$$(\mathbf{t}(s), \mathbf{u}(s), \mathbf{n}(s))$$

$$t' = r \times t$$

$$u' = r \times u$$

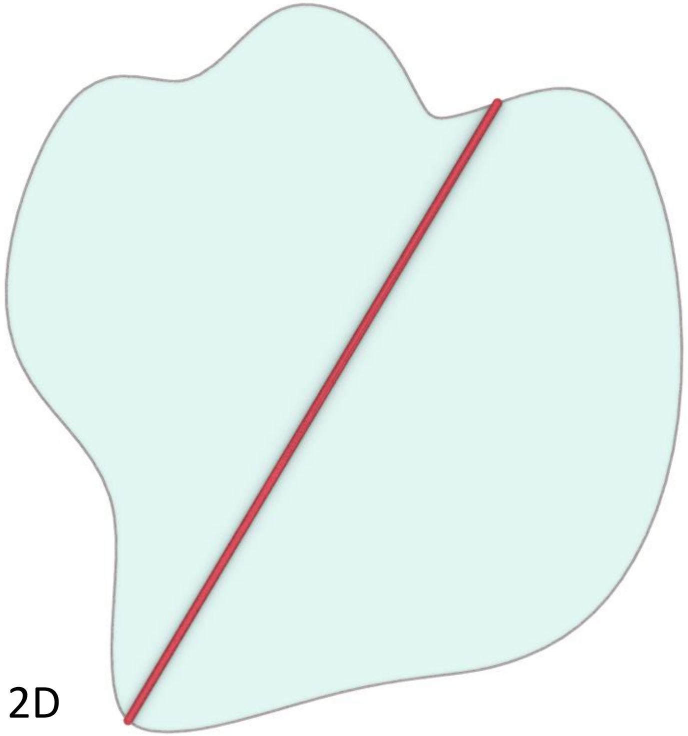
$$n' = r \times n$$

Angular velocity vector:

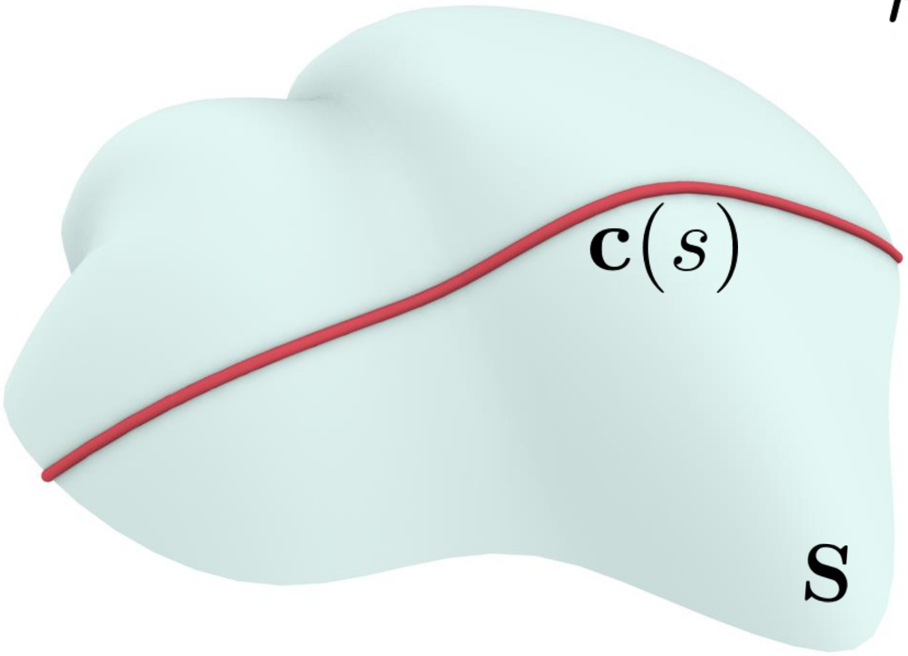
$$r = \tau_g t - \kappa_n u + \kappa_g n$$

Geodesic curve

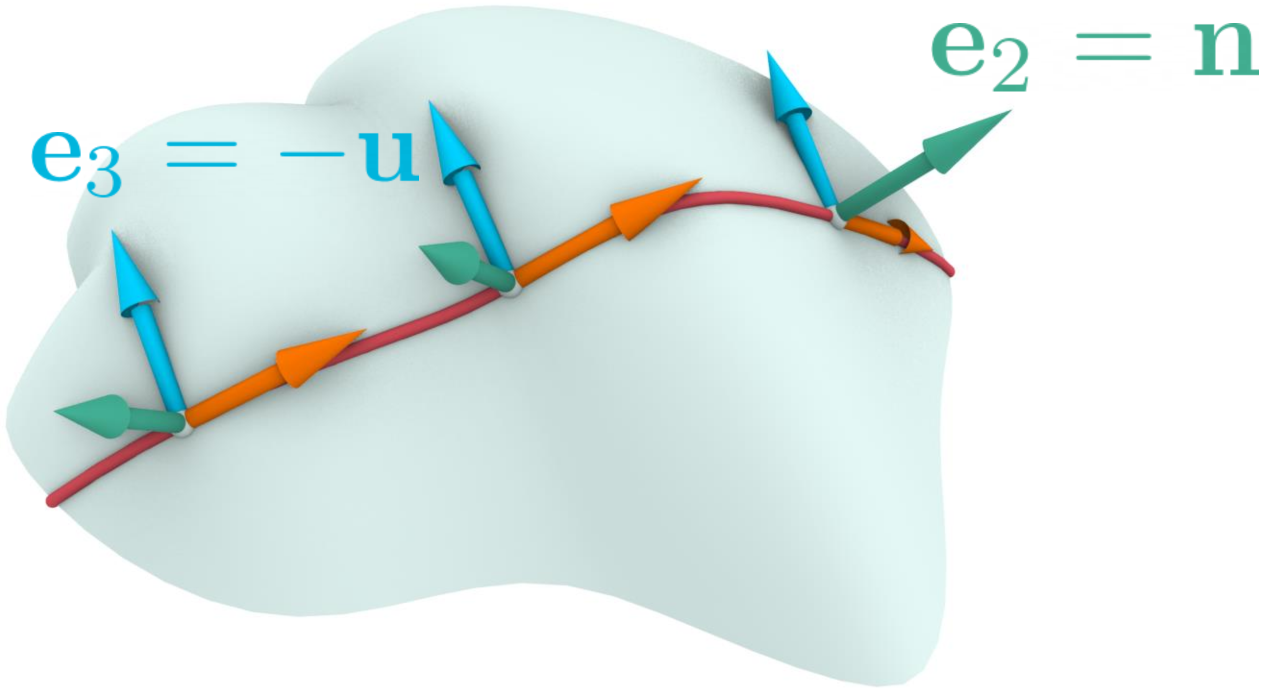
$$\kappa_g = 0$$



2D



Geodesic curve



$$\kappa_g = 0$$

$$\mathbf{c}'' = \kappa_n \mathbf{n}$$

$$\mathbf{e}_2 = \mathbf{n}, \mathbf{e}_3 = -\mathbf{u}$$

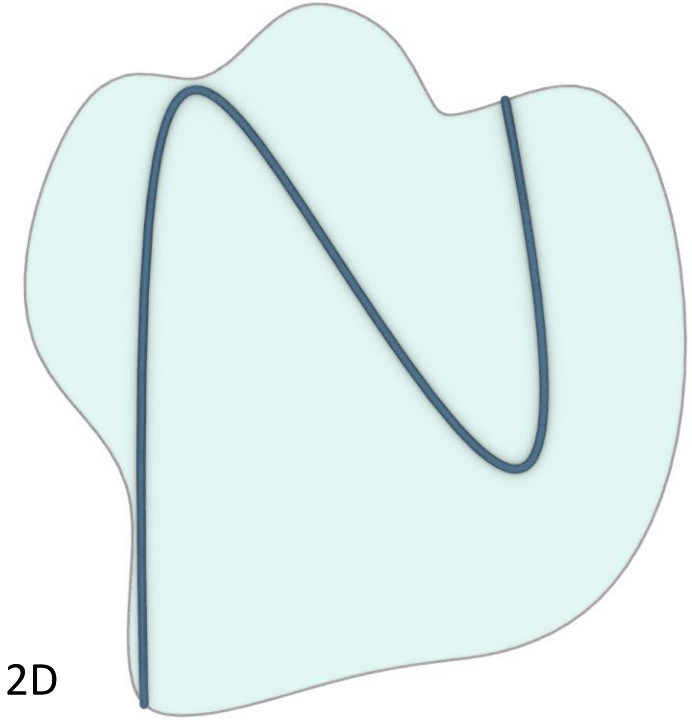
Geodesic strip

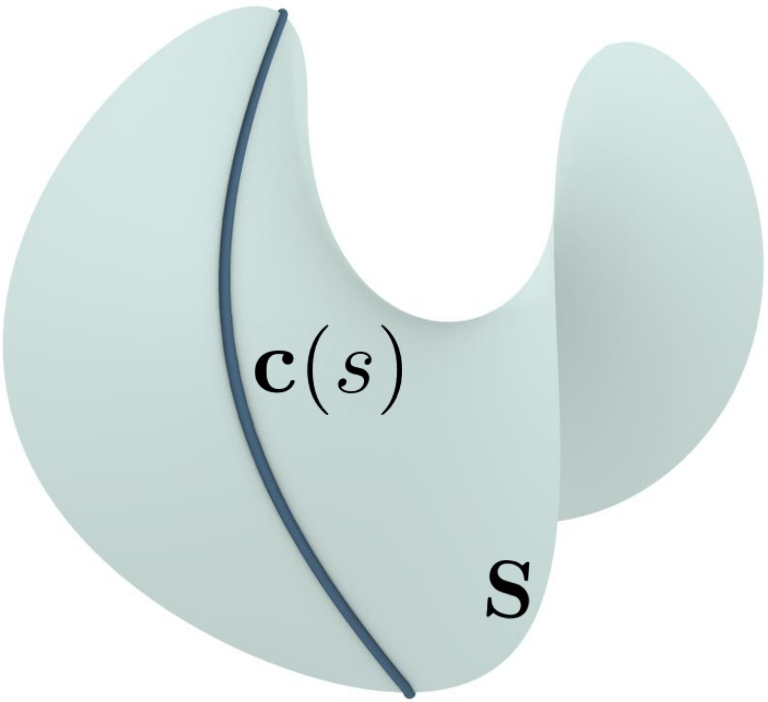
$$\kappa_g = 0$$

$$\mathbf{c}'' = \kappa_n \mathbf{n}$$

$$\mathbf{e}_2 = \mathbf{n}, \mathbf{e}_3 = -\mathbf{u}$$

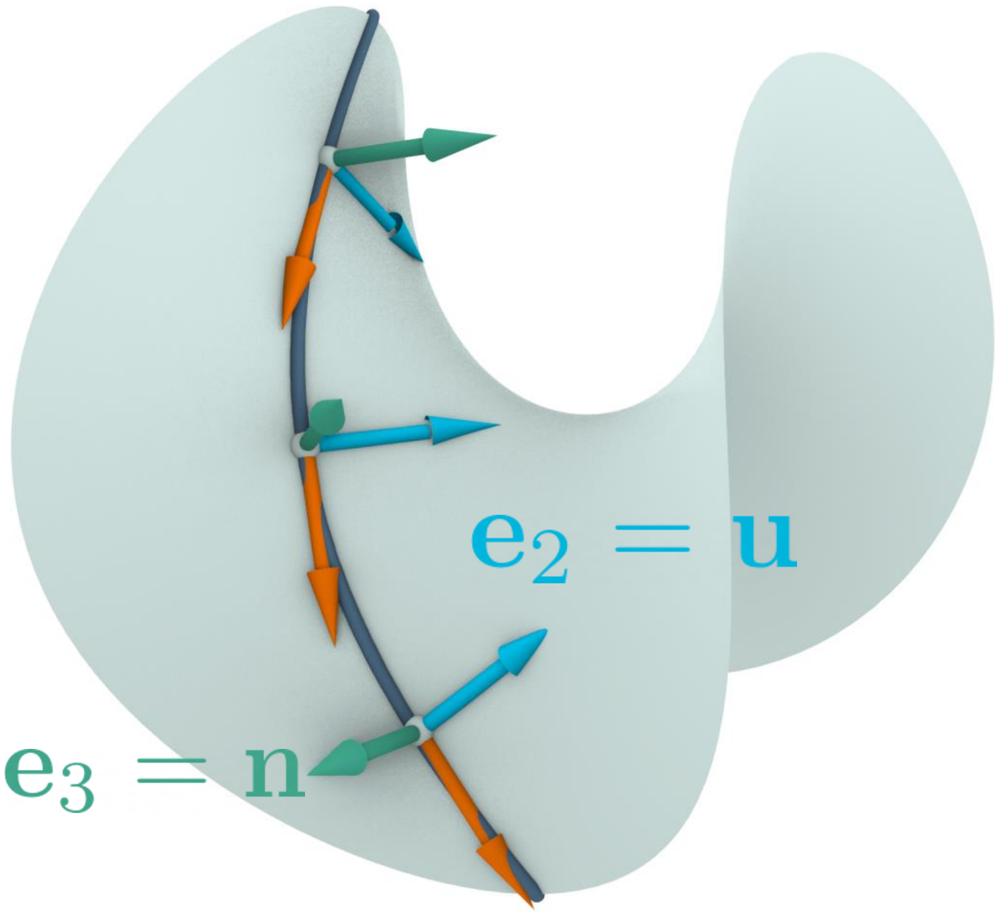
Asymptotic curve





$$\kappa_n = 0$$

Asymptotic curve

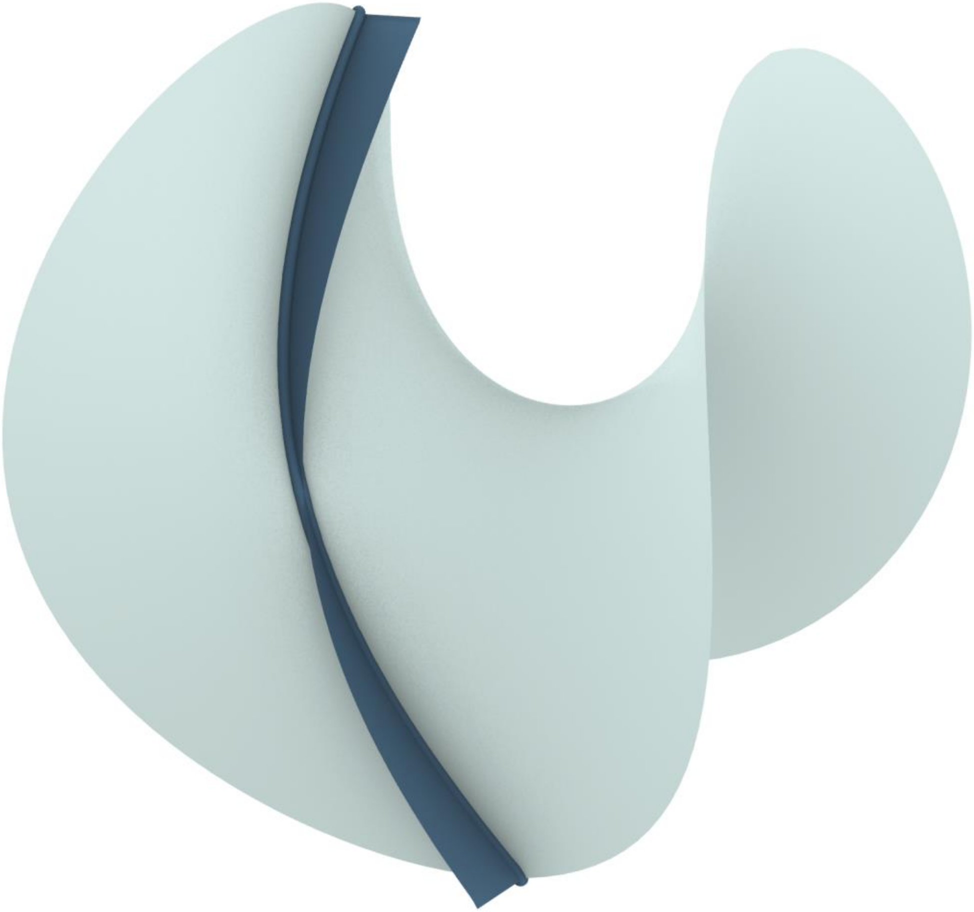


$$\kappa_n = 0$$

$$\mathbf{c}'' = \kappa_g \mathbf{u}$$

$$\mathbf{e}_2 = \mathbf{u}, \mathbf{e}_3 = \mathbf{n}$$

Asymptotic strip

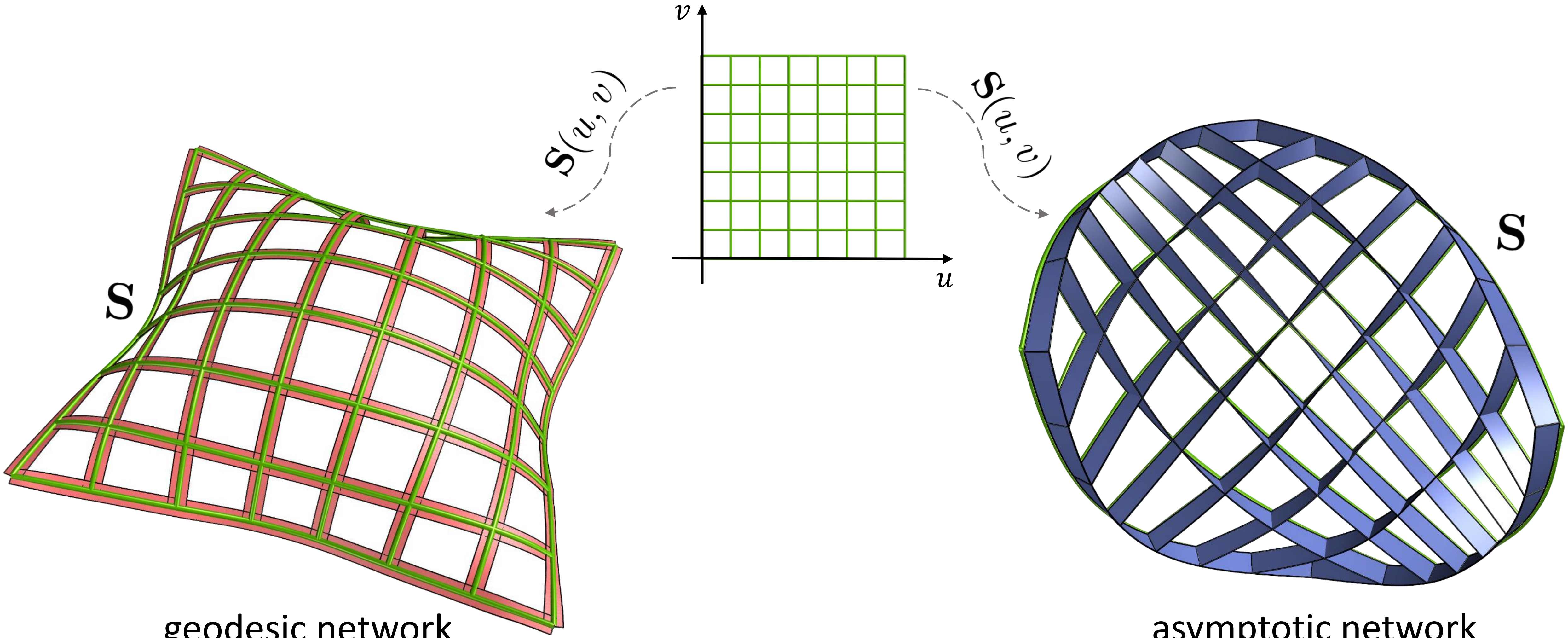


$$\kappa_n = 0$$

$$\mathbf{c}'' = \kappa_g \mathbf{u}$$

$$\mathbf{e}_2 = \mathbf{u}, \mathbf{e}_3 = \mathbf{n}$$

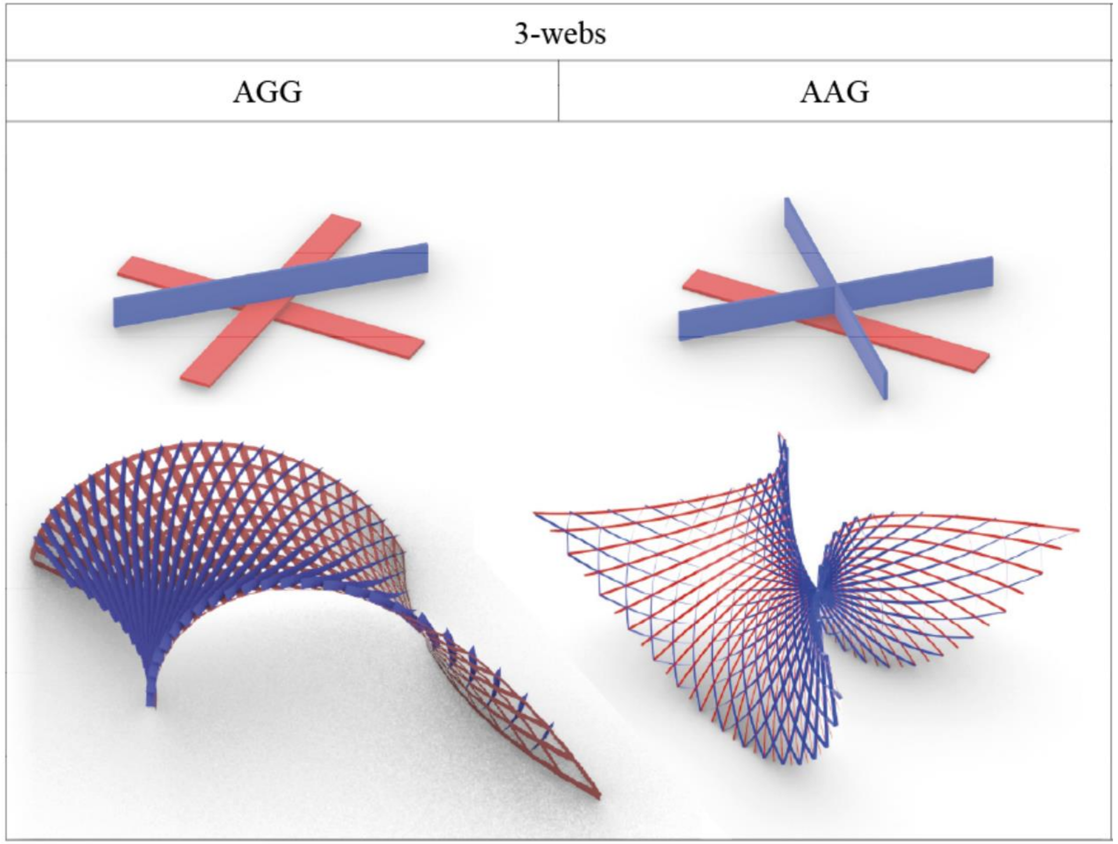
Surface parametrization

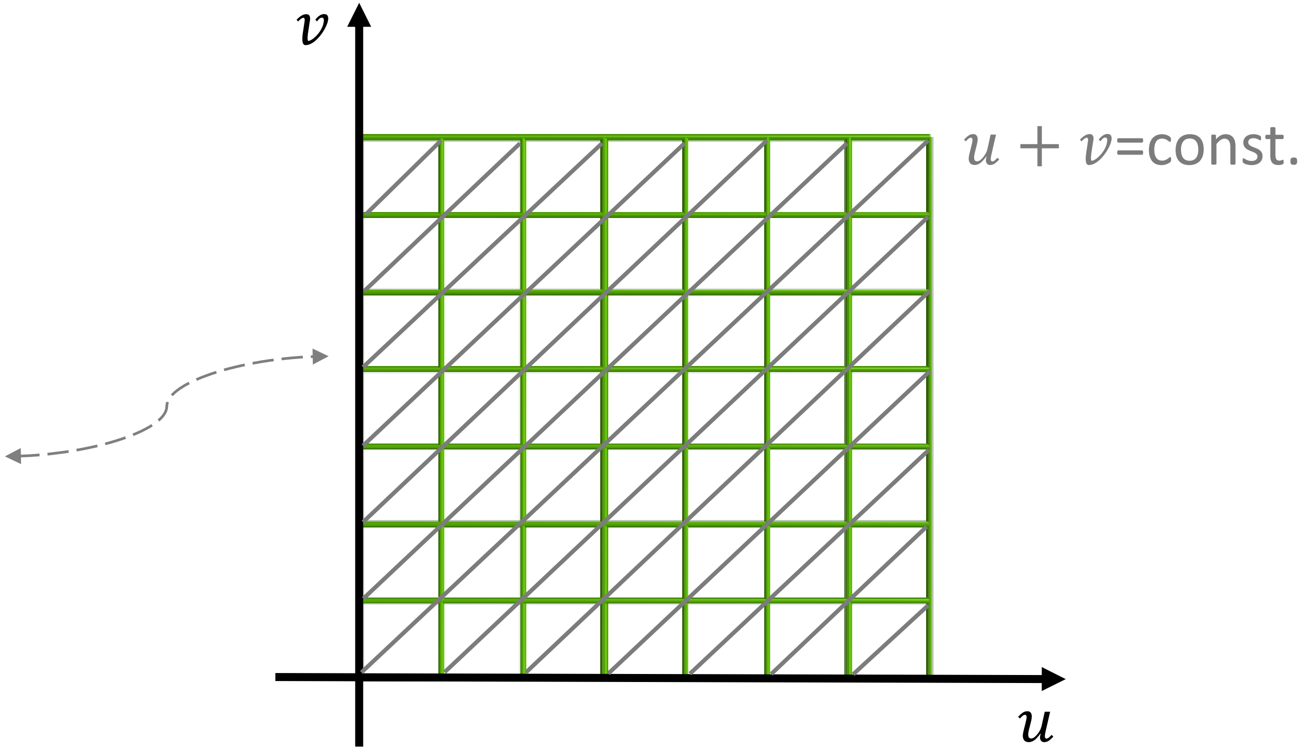


geodesic network

asymptotic network

AGG-,AAG-web

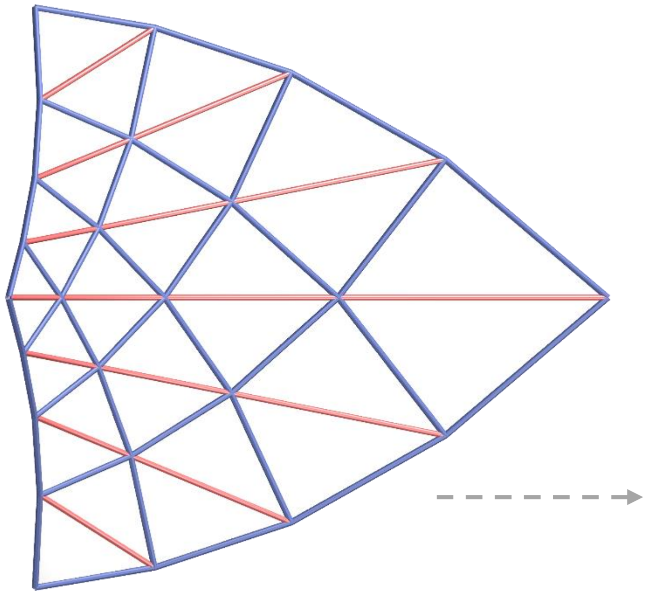




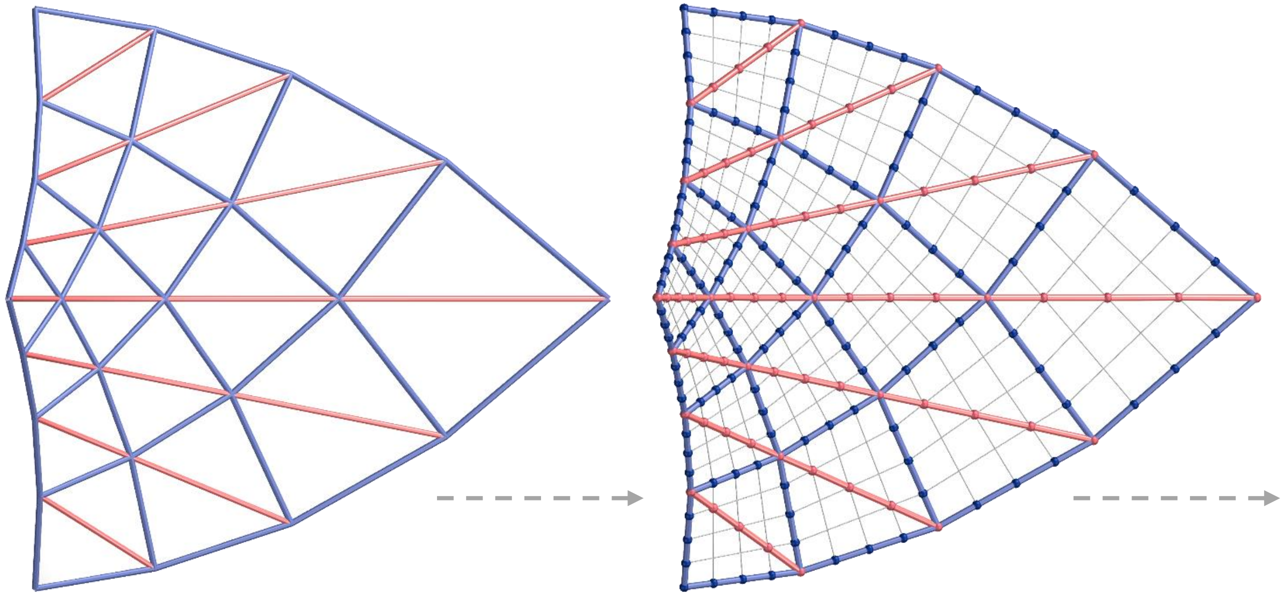
- Geodesic
- Asymptote

Discretization and optimization

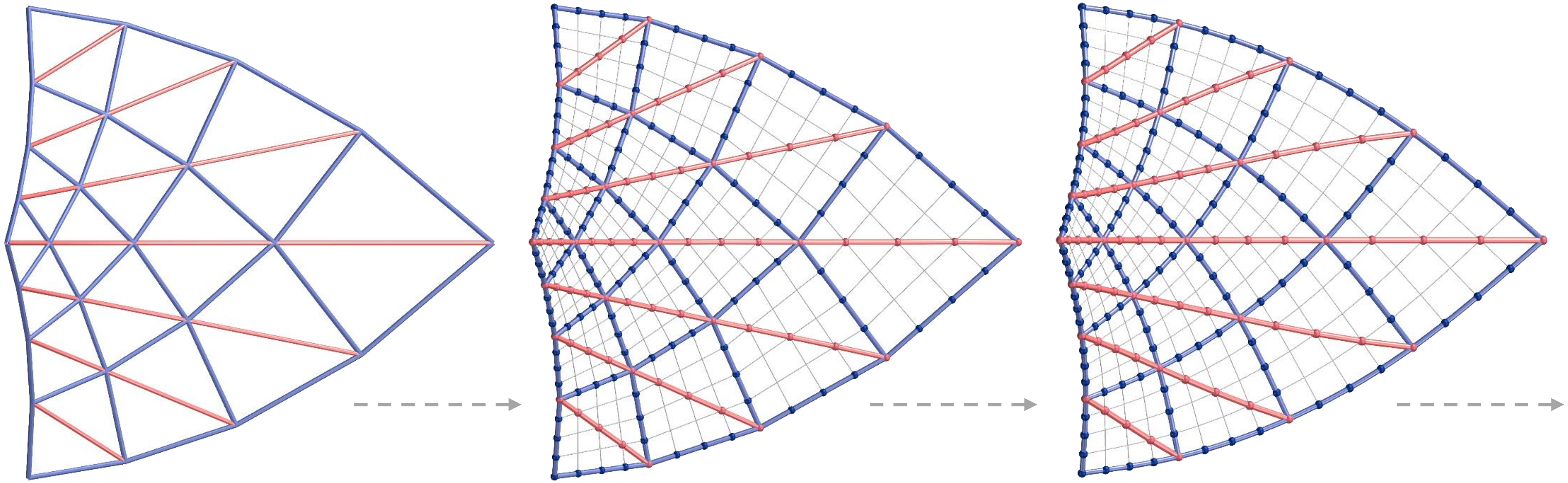
Computational approach



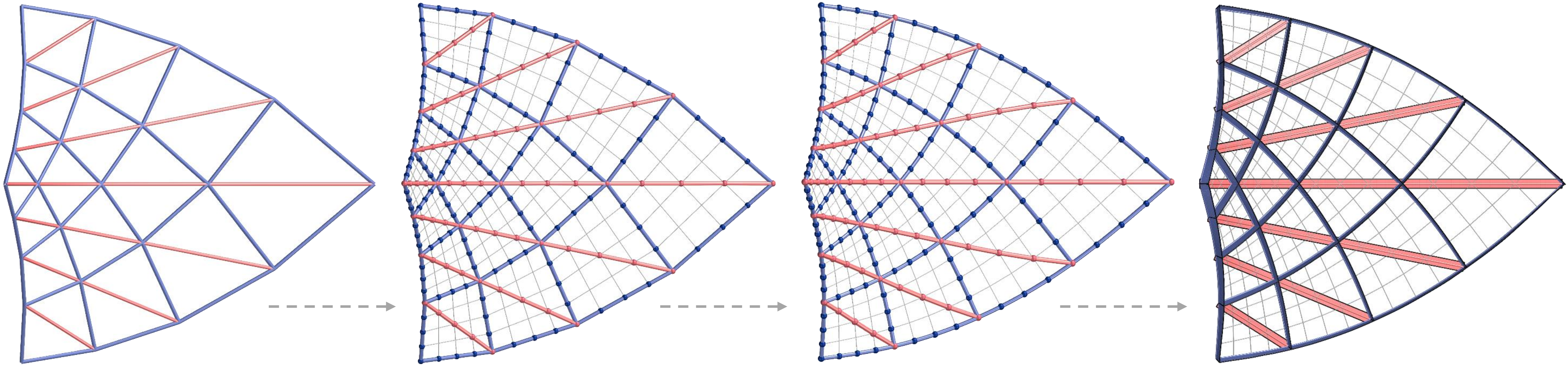
Computational approach



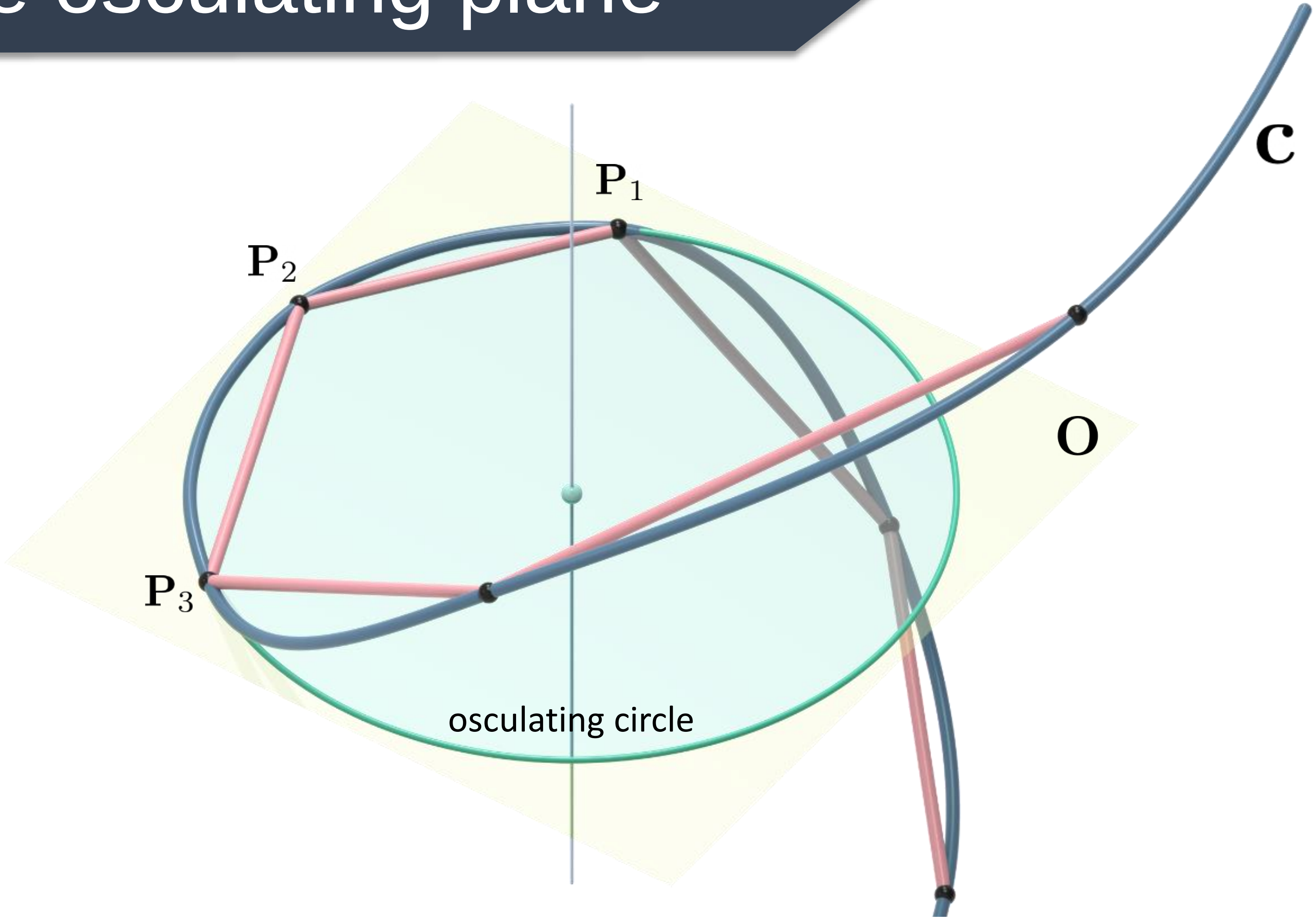
Computational approach



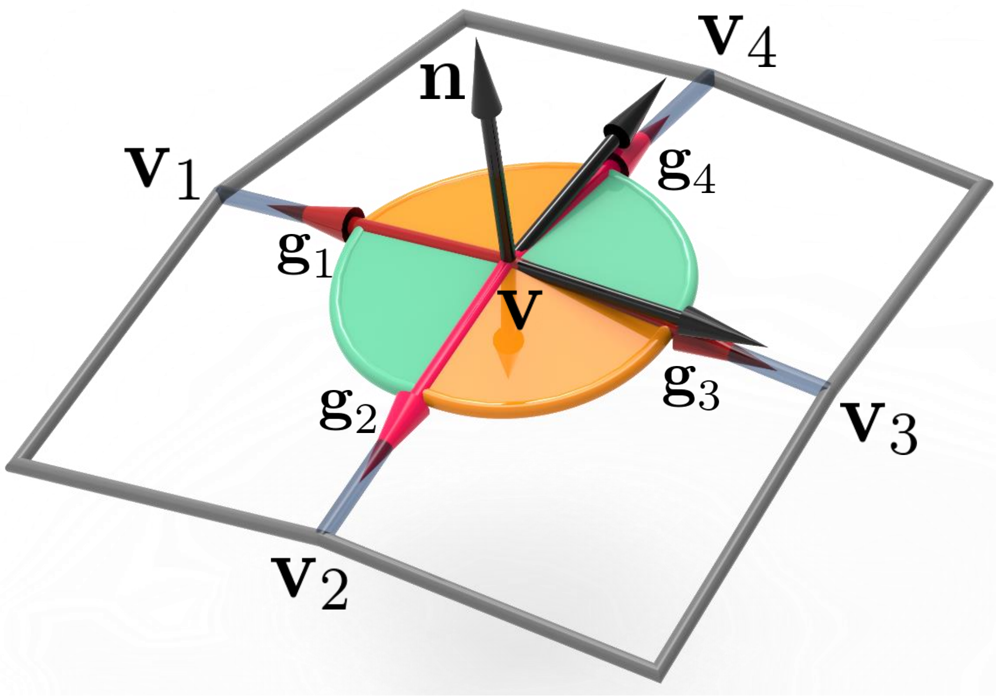
Computational approach



Discrete osculating plane



Constraints: G-net



[Wunderlich 1951, Rabinovich et al. 2018]

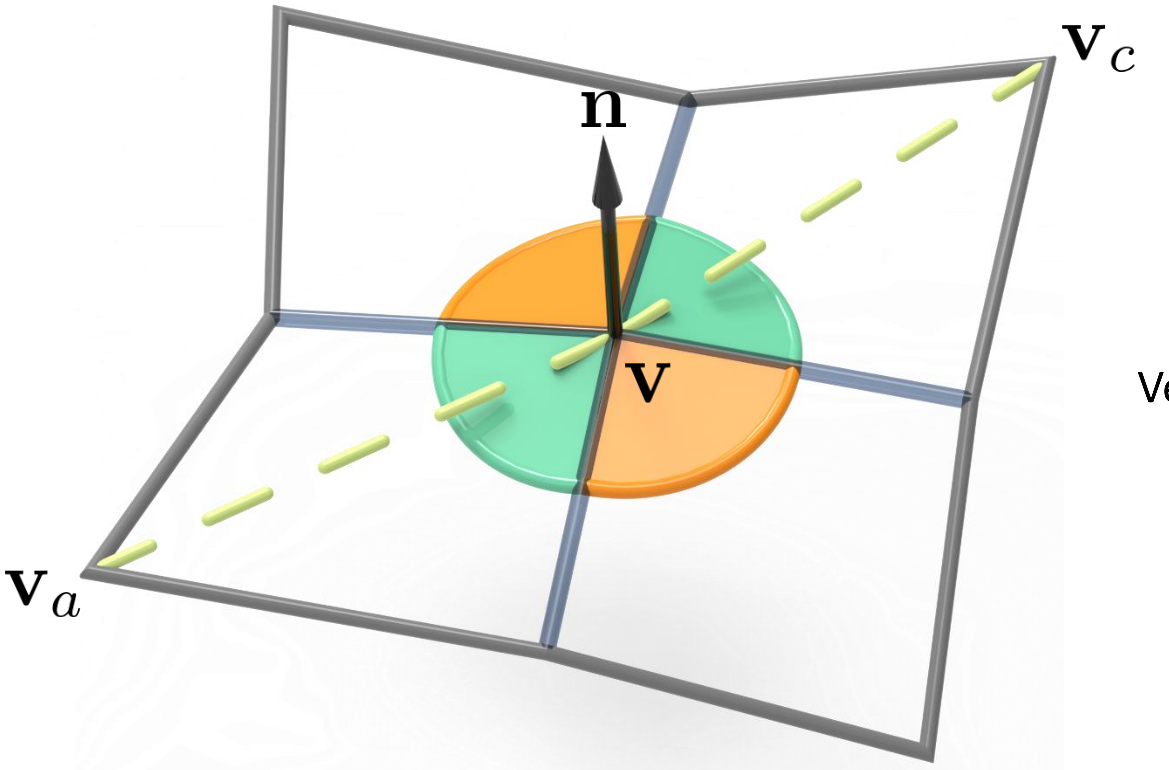
$$\mathbf{g}_i = (\mathbf{v}_i - \mathbf{v})/\|\mathbf{v}_i - \mathbf{v}\|, \quad i = 1, \ldots, 4,$$

$$\mathbf{g}_1 \cdot \mathbf{g}_2 = \mathbf{g}_3 \cdot \mathbf{g}_4, \ \mathbf{g}_2 \cdot \mathbf{g}_3 = \mathbf{g}_4 \cdot \mathbf{g}_1.$$

Vertex normal:

$$\mathbf{n} \mid\mid \mathbf{g}_1 + \mathbf{g}_3 \mid\mid \mathbf{g}_2 + \mathbf{g}_4$$

Constraints: AGG-web



$$\mathbf{g}_i = (\mathbf{v}_i - \mathbf{v})/\|\mathbf{v}_i - \mathbf{v}\|, \quad i = 1, \ldots, 4,$$

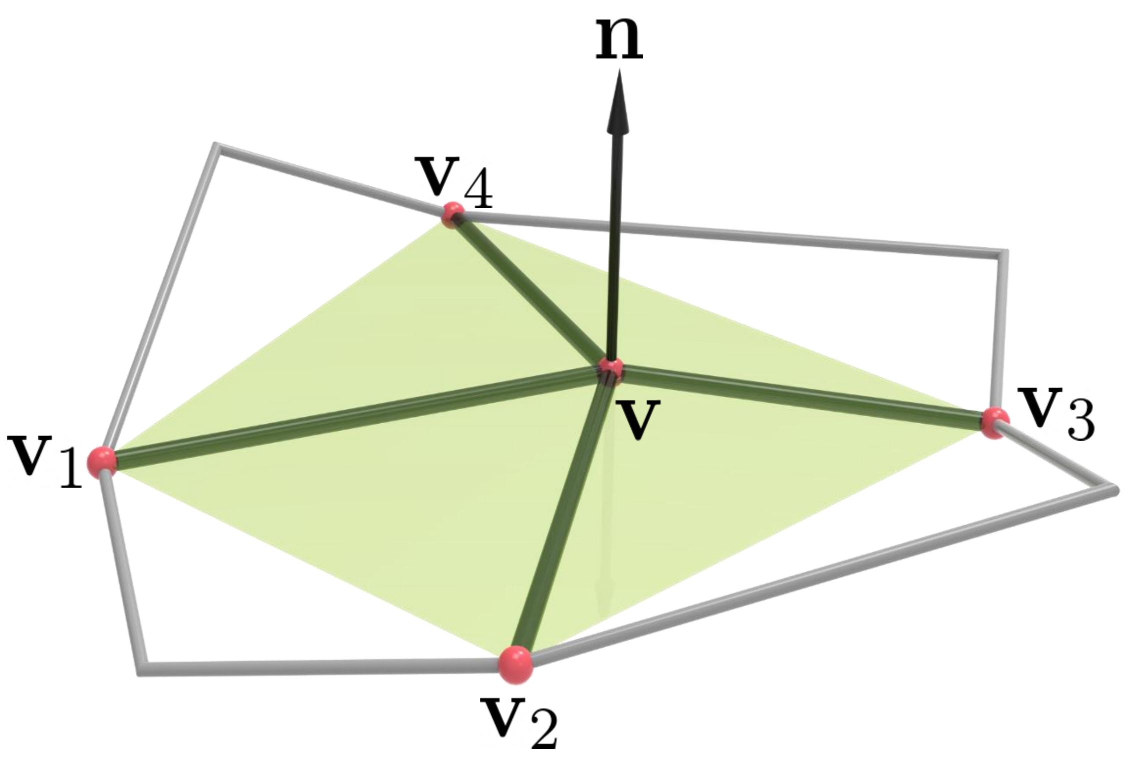
$$\mathbf{g}_1 \cdot \mathbf{g}_2 = \mathbf{g}_3 \cdot \mathbf{g}_4, \ \mathbf{g}_2 \cdot \mathbf{g}_3 = \mathbf{g}_4 \cdot \mathbf{g}_1.$$

Vertex normal:

$$\mathbf{n} \parallel \mathbf{g}_1 + \mathbf{g}_3 \parallel \mathbf{g}_2 + \mathbf{g}_4$$

$$\mathbf{n} \cdot (\mathbf{v}_a - \mathbf{v}) = 0, \ \mathbf{n} \cdot (\mathbf{v}_c - \mathbf{v}) = 0.$$

Constraints: A-net

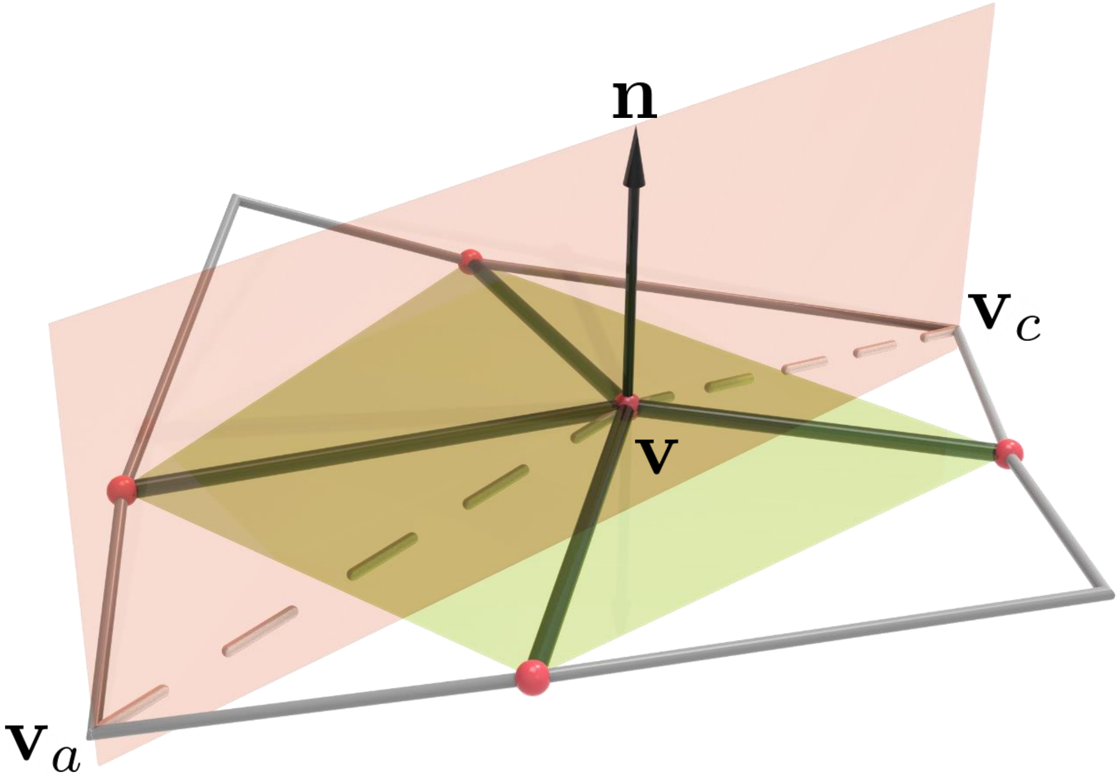


[Bobenko et al. 2008]

$$\mathbf{n} \cdot (\mathbf{v}_i - \mathbf{v}) = 0, \qquad i = 1, \ldots, 4.$$

$$\|\mathbf{n}\|^2 = 1.$$

Constraints: AAG-web

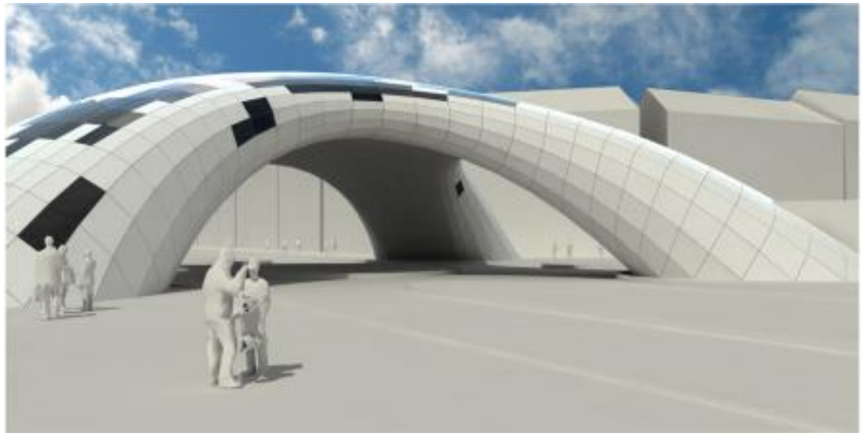


$$\mathbf{n} \cdot (\mathbf{v}_i - \mathbf{v}) = 0, \qquad i = 1, \ldots, 4.$$

$$\|\mathbf{n}\|^2 = 1.$$

$$\boxed{\mathbf{n} \cdot [(\mathbf{v}_a - \mathbf{v}) \times (\mathbf{v}_c - \mathbf{v})] = 0.}$$

Target function



[Tang et al. 2014]

$$F(X) = \sum (C_i(X)^2 + \omega_1 K(X)^2 + \omega_2 V(X)^2 + \omega_3 P(X)^2 + \varepsilon (X - X_c)^2)$$

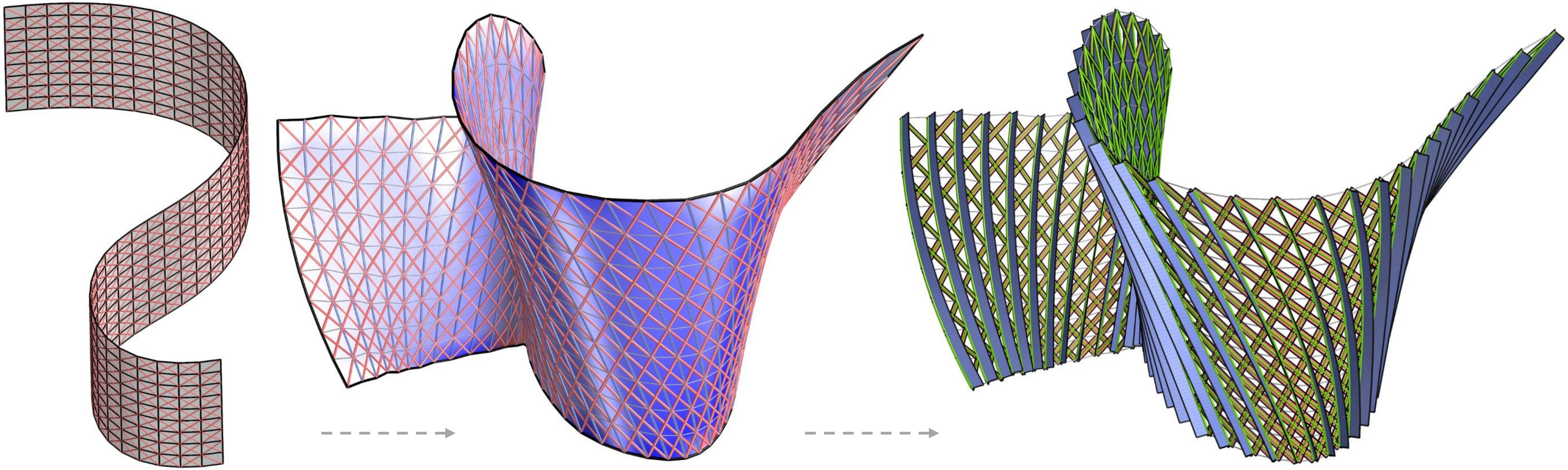
Fairness: $\mathbf{v}_{i+1} - 2\mathbf{v}_i + \mathbf{v}_{i-1} = 0$

Proximity to curve or surfaces: $(\mathbf{v}_p - \mathbf{p}) \cdot \mathbf{n}_p = 0$

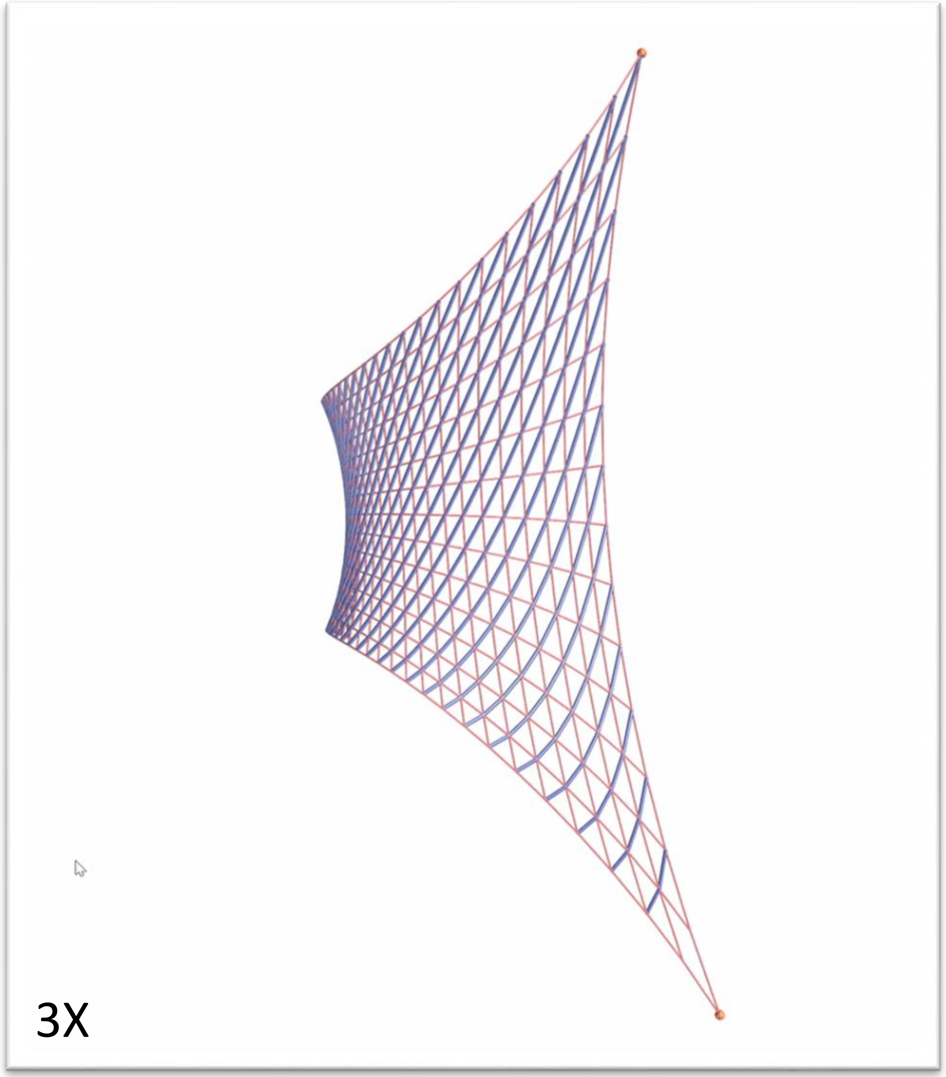
Control the change of vertices: $\mathbf{v}^{(j-1)} - \mathbf{v}^{(j)} = 0$

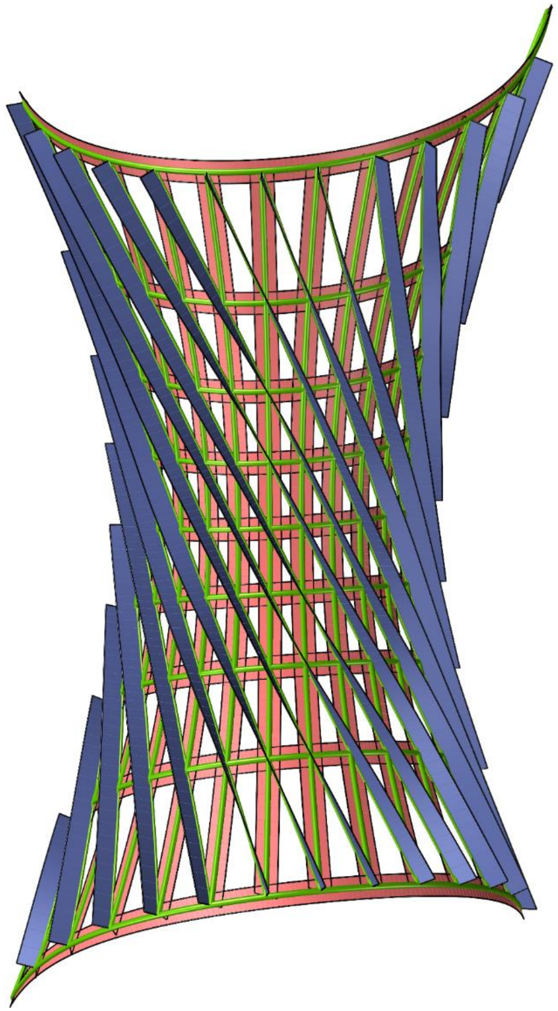
$\varepsilon = 0.001$

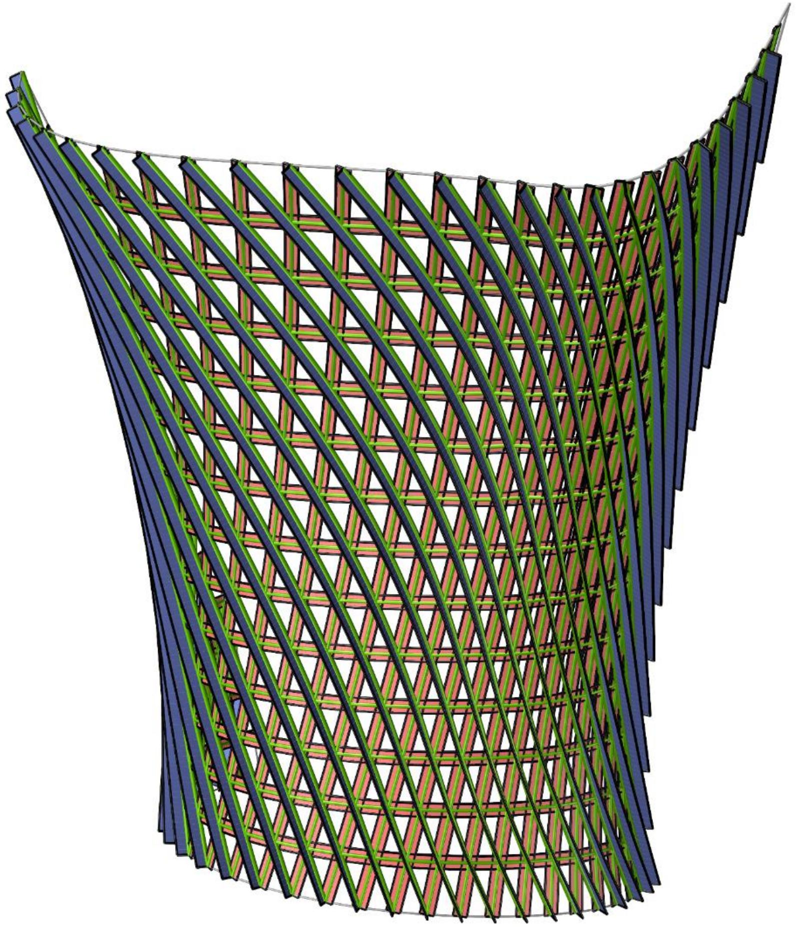
Initialization: AGG-web



AGG-webs

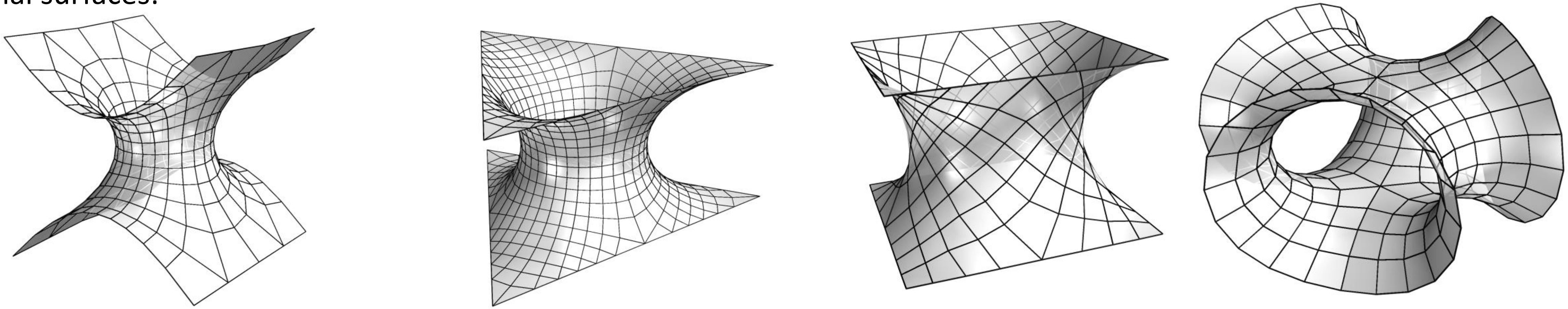






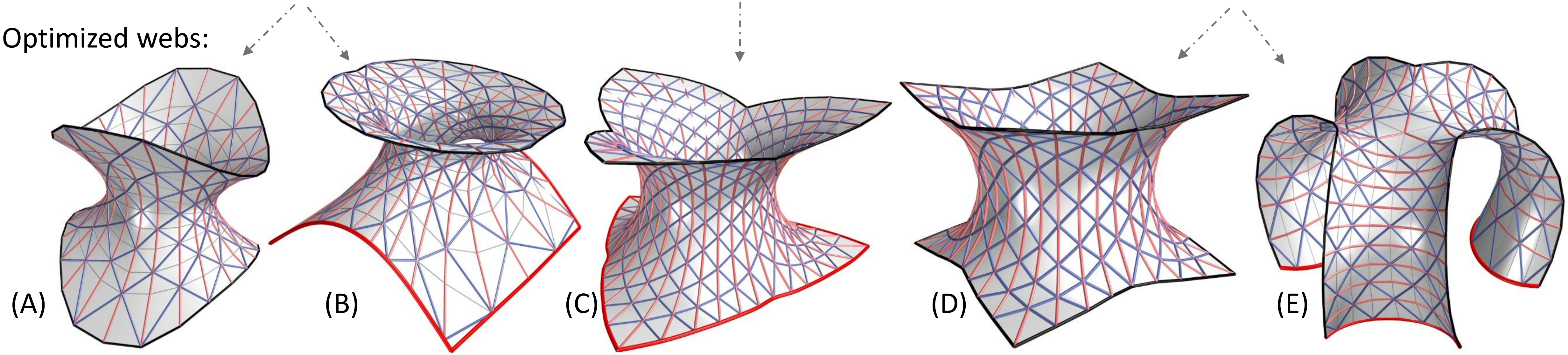
Initialization: AAG-web

Initial minimal surfaces:

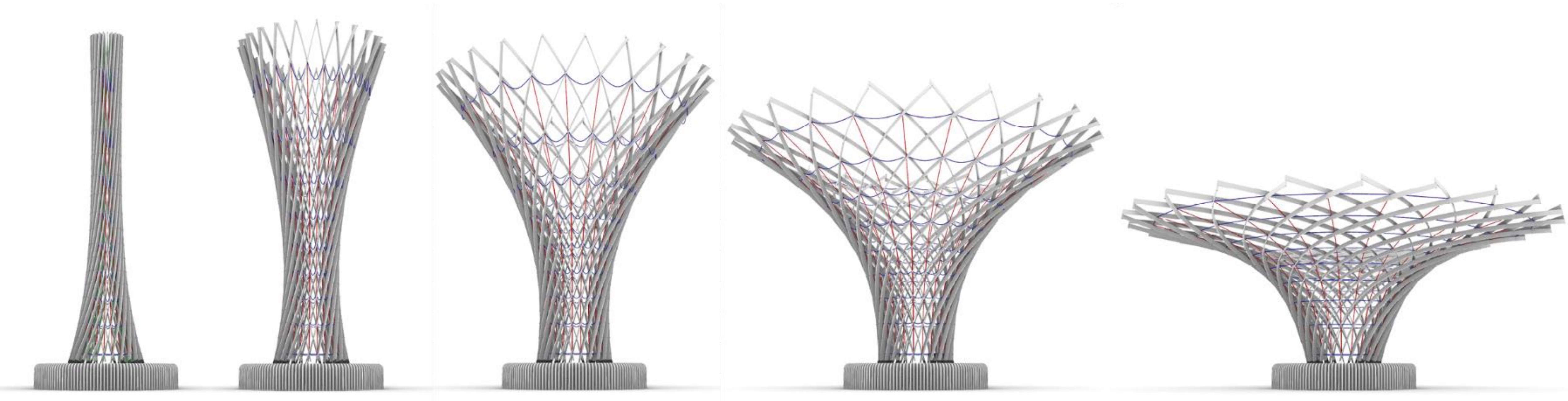


Scherk tower surface   Schwarz H surface   Schwarz P surface

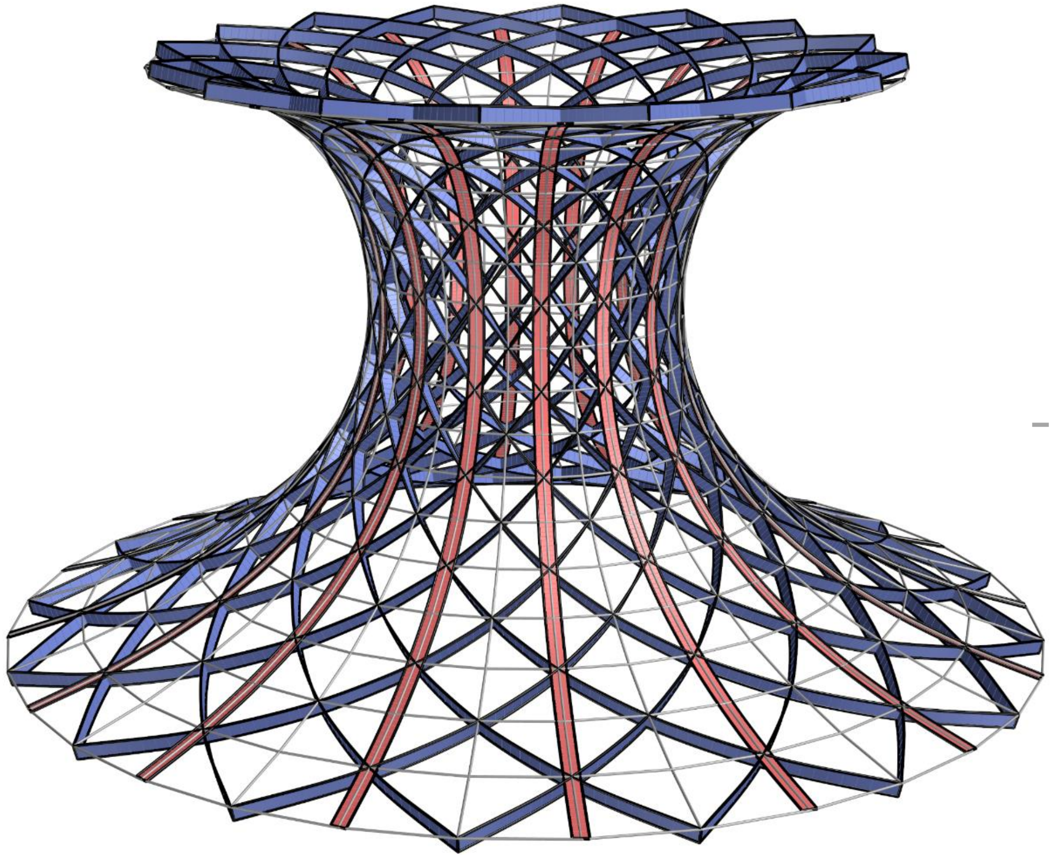
Optimized webs:

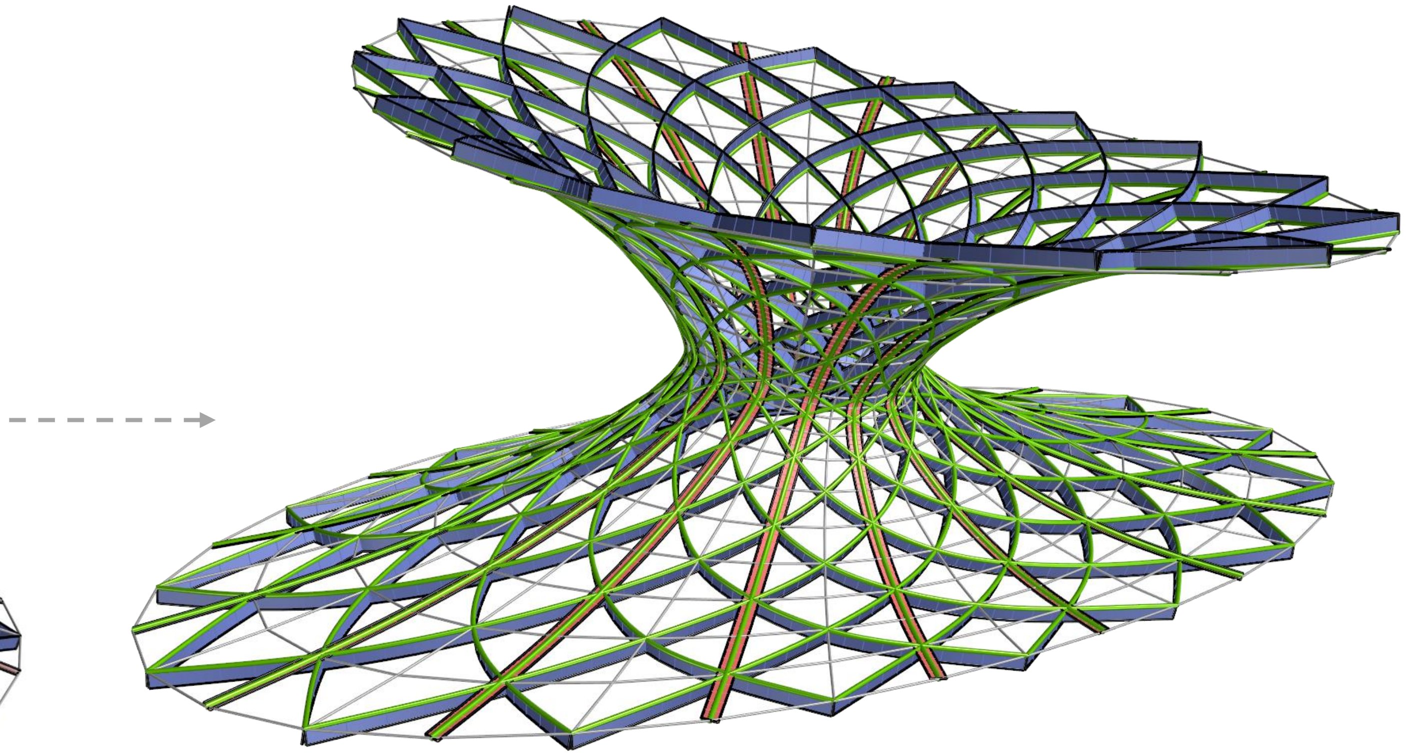


AAG-web

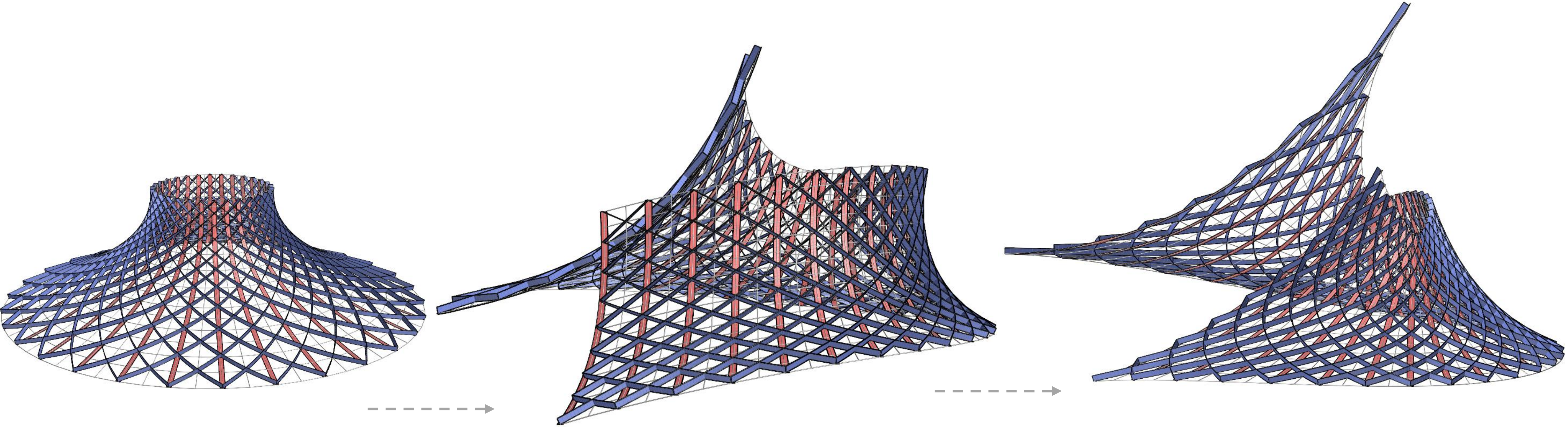


Initialization: AAG-web

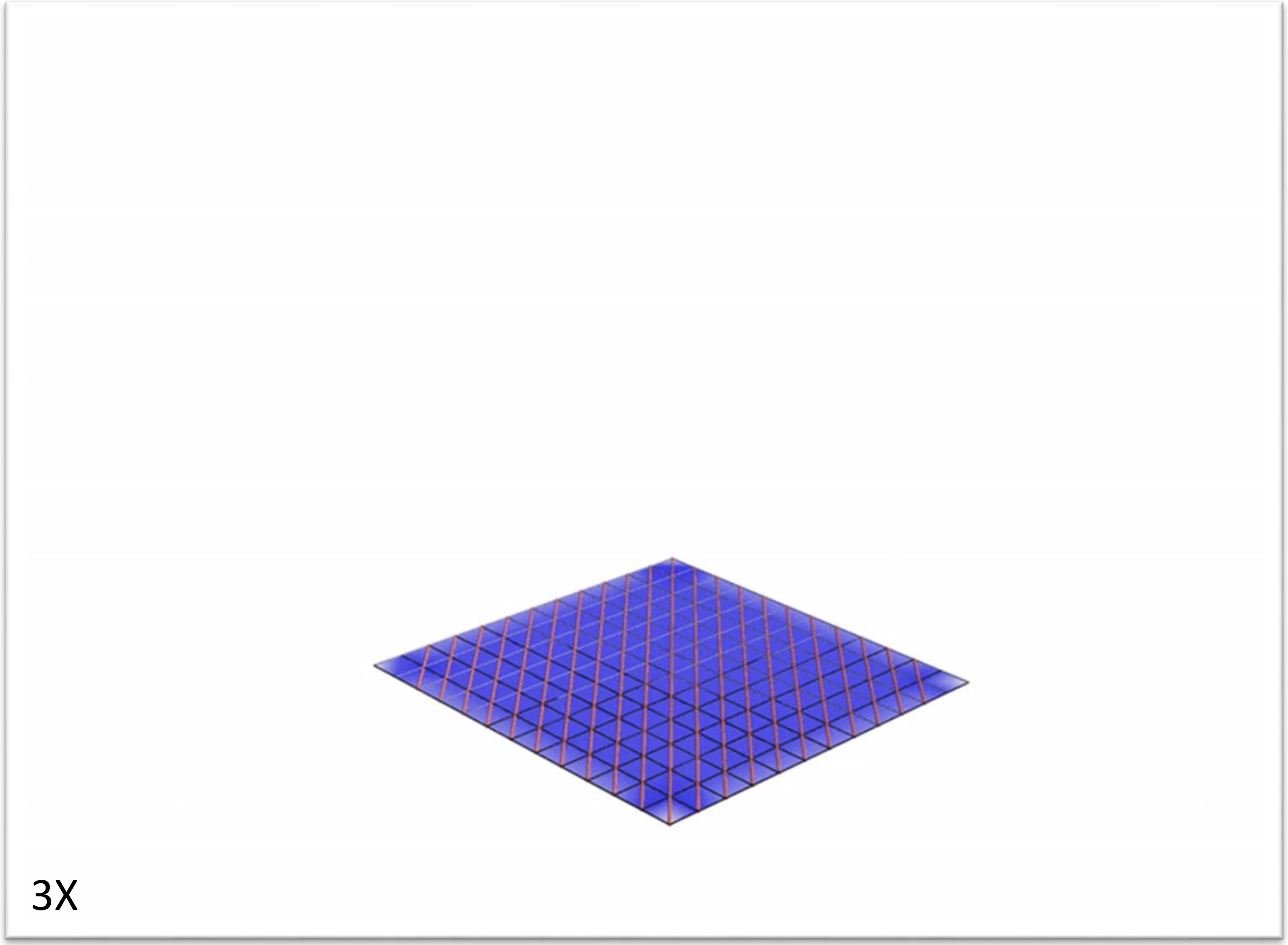




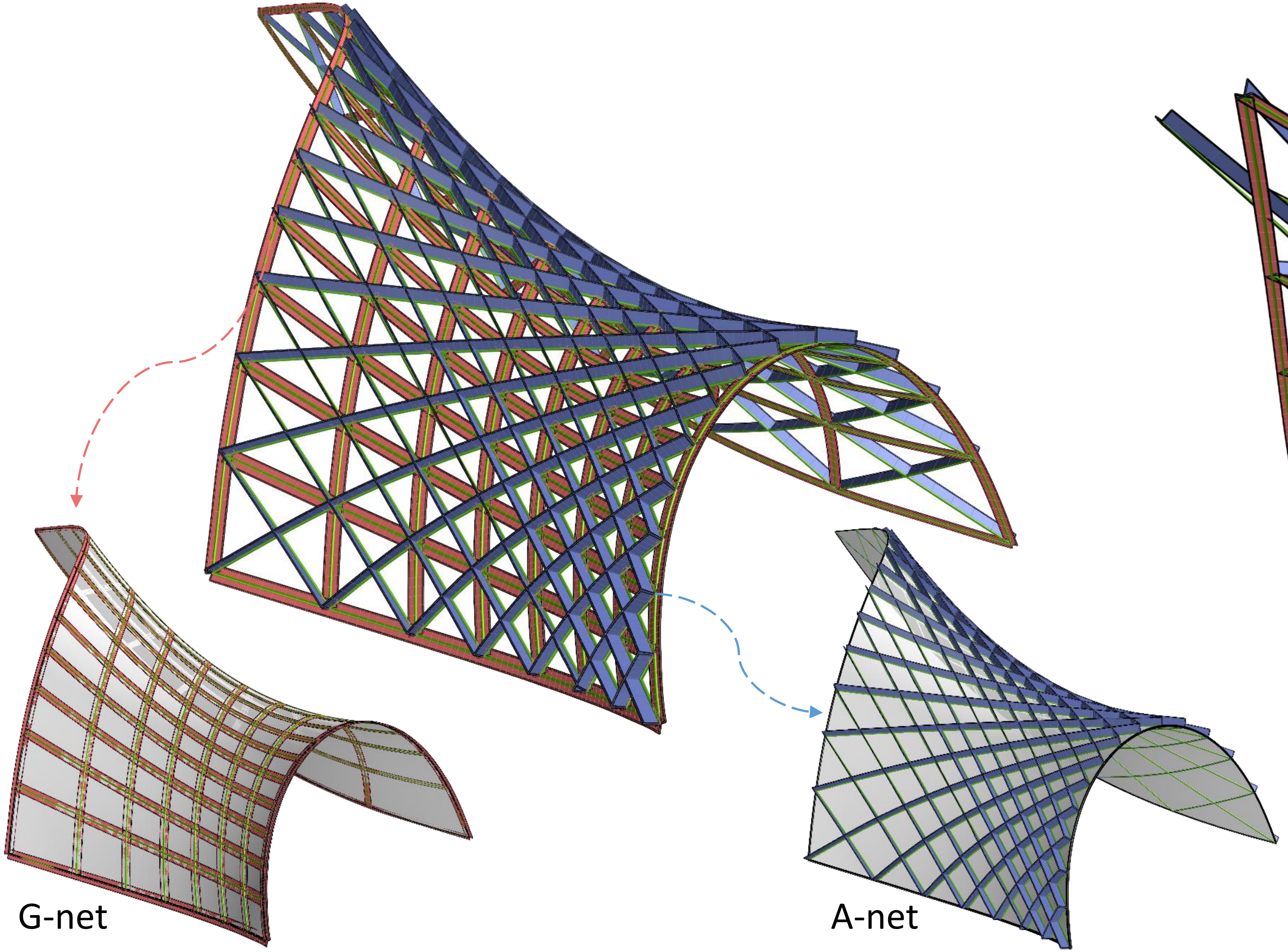
Initialization: AAG-web



AAG-webs

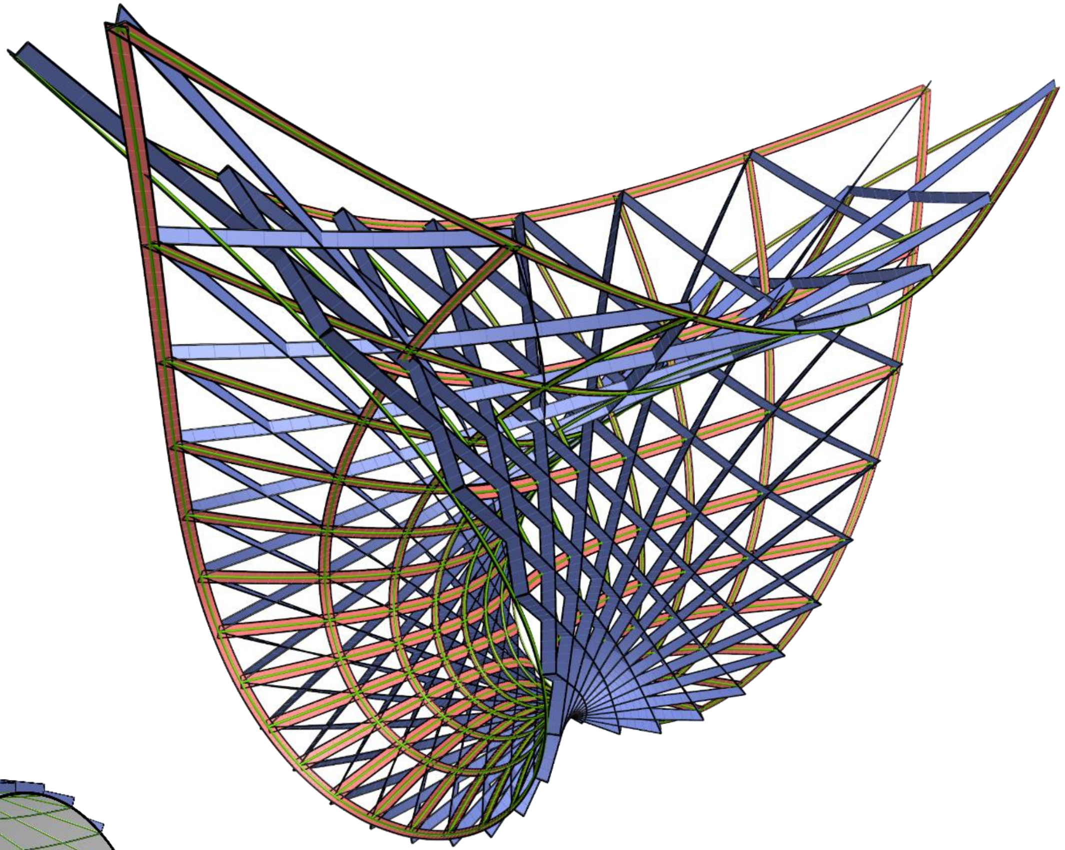


AGAG-webs

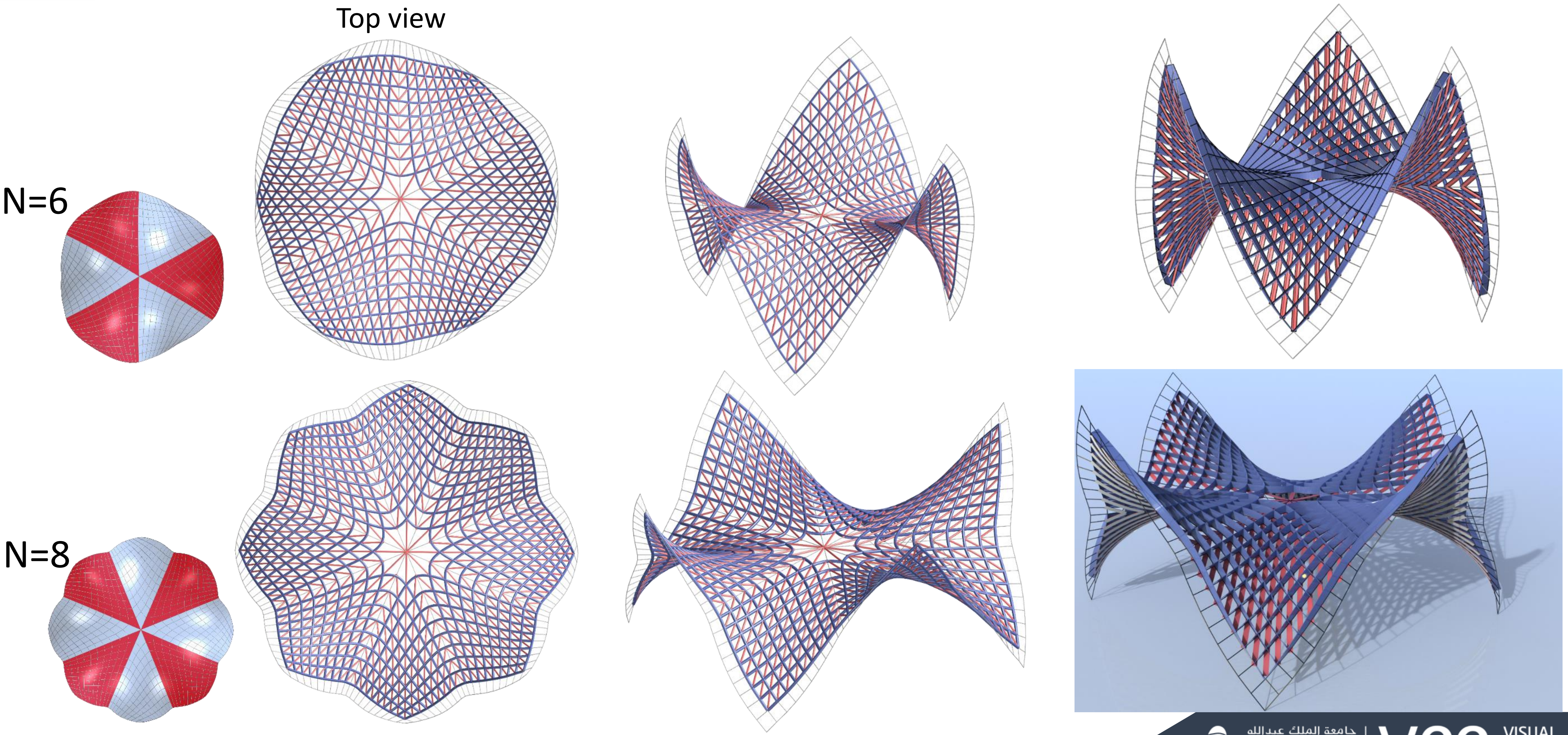


G-net

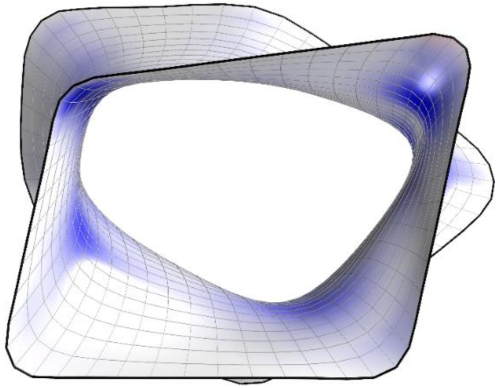
A-net



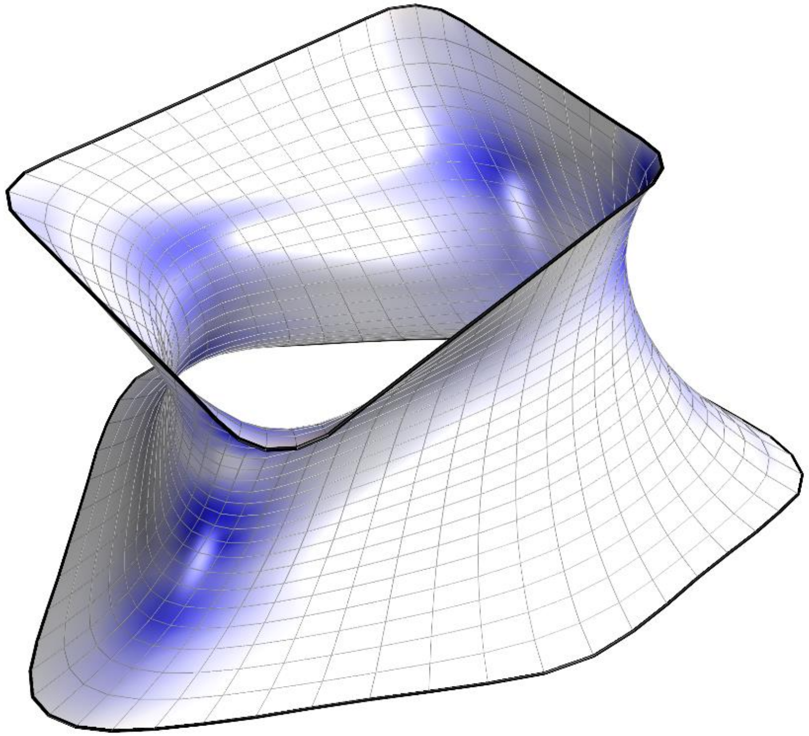
With combinatorial singularity



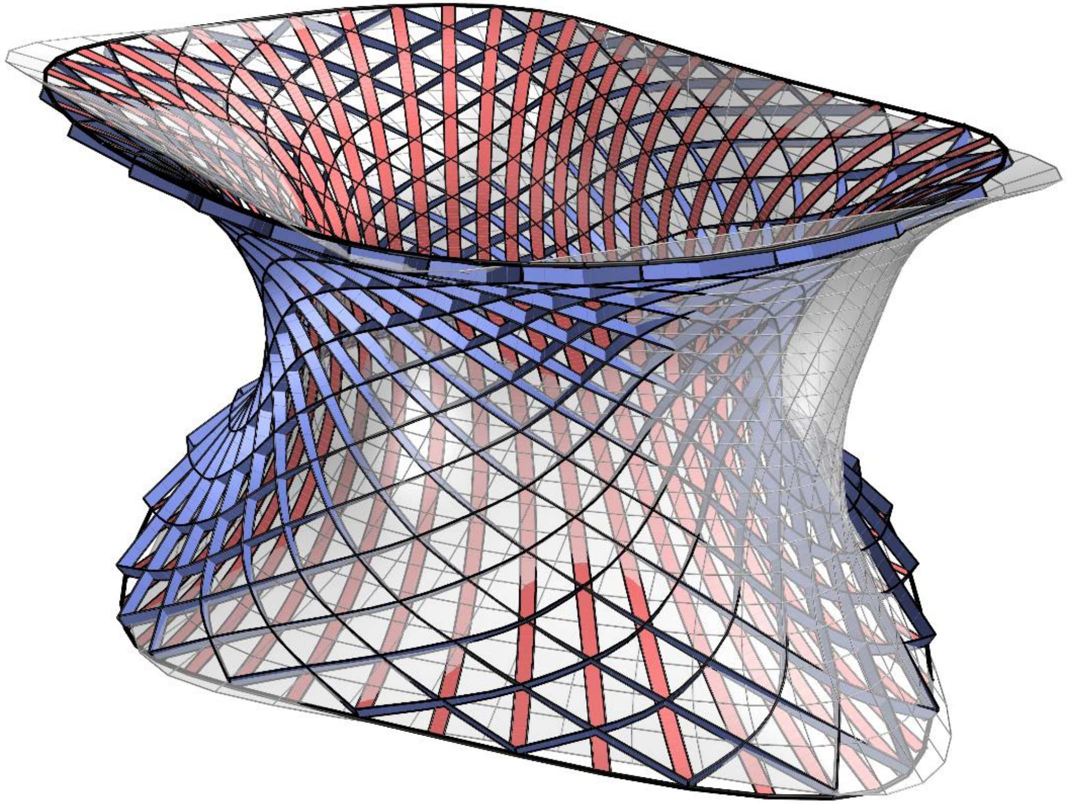
Shape implementation



Top view



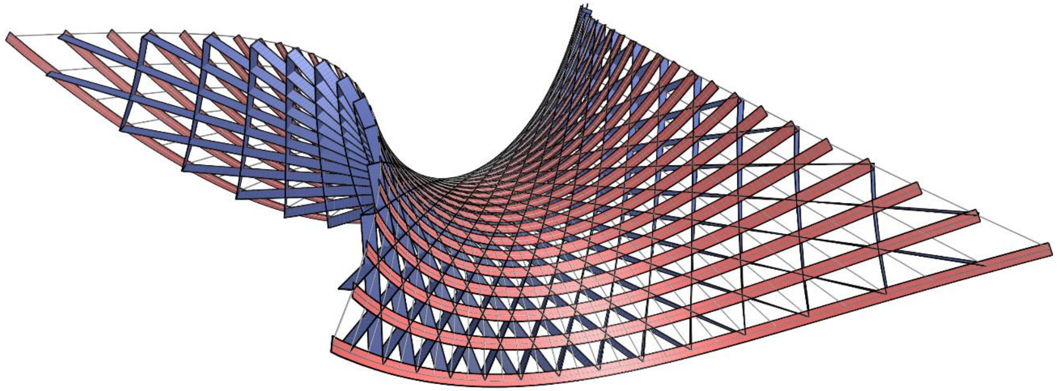
Soumaya Museum

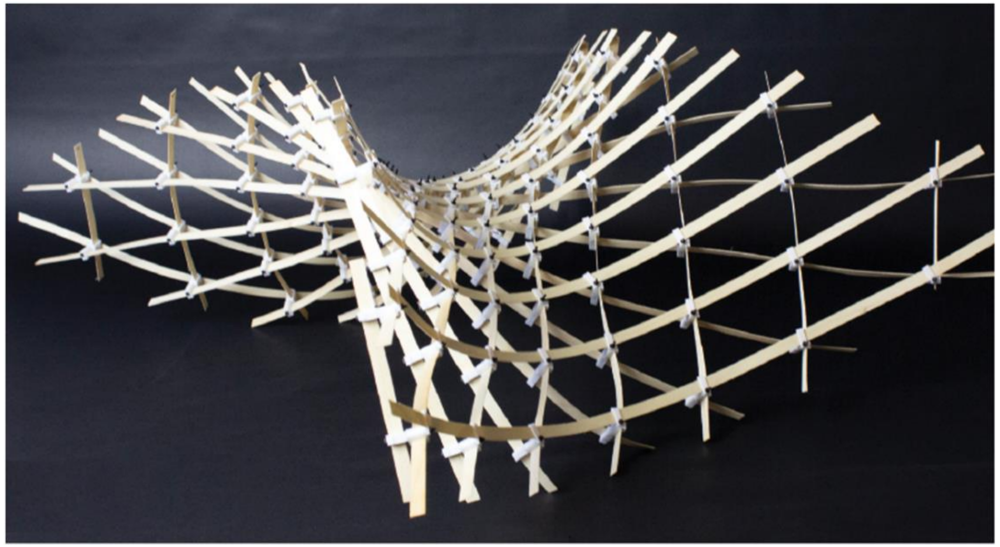


AAG web roughly approximates the surface

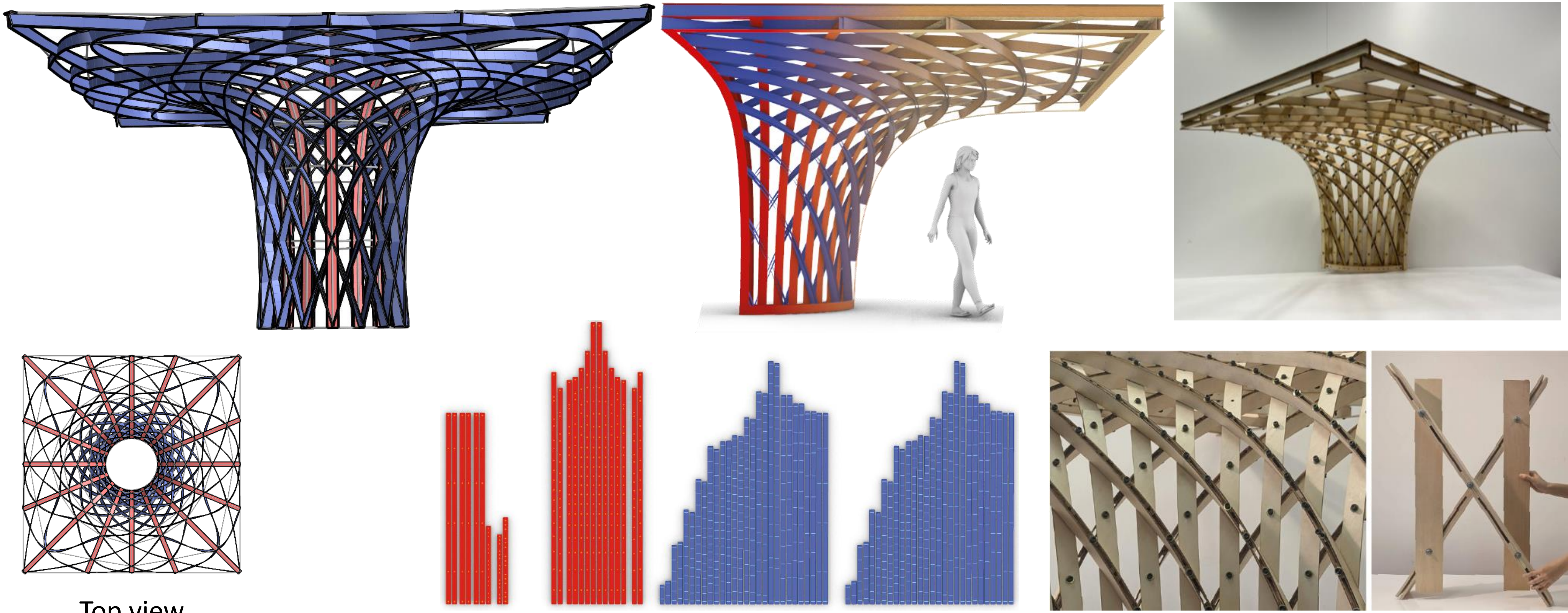
Application

AAG gridshell



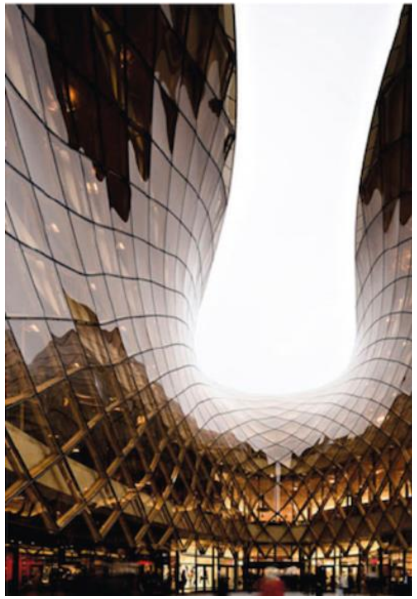


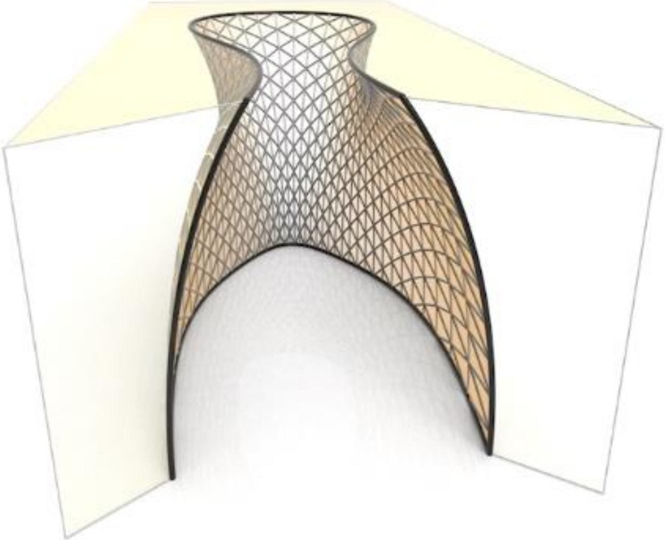
AAG gridshell



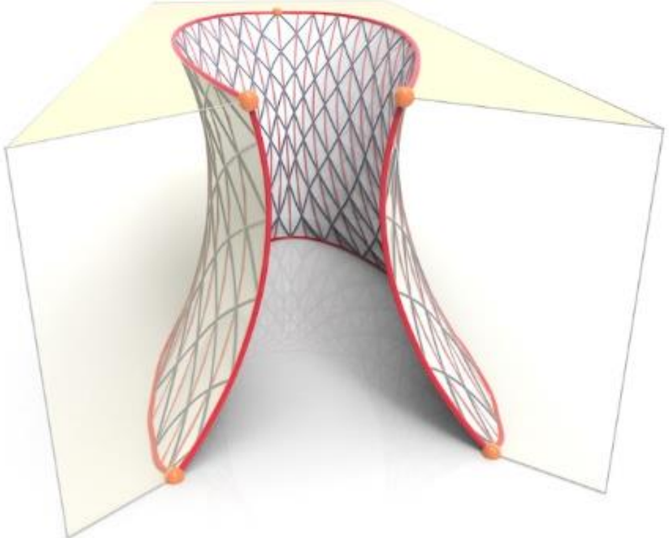
Top view

AAG gridshell

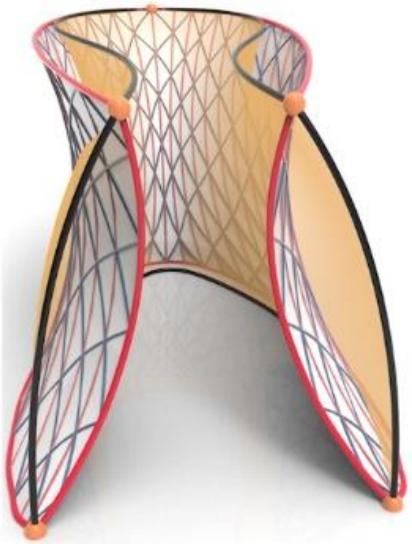




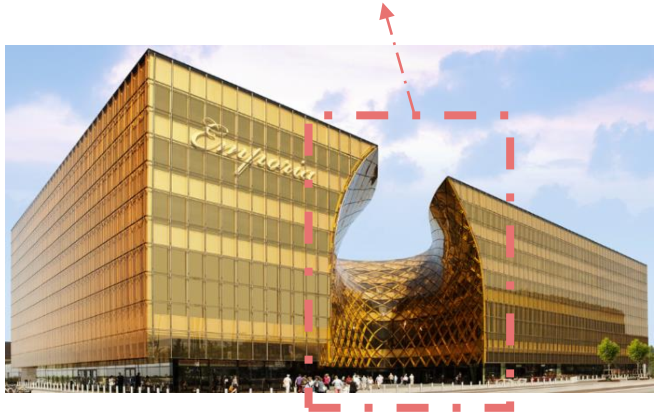
Initial mesh



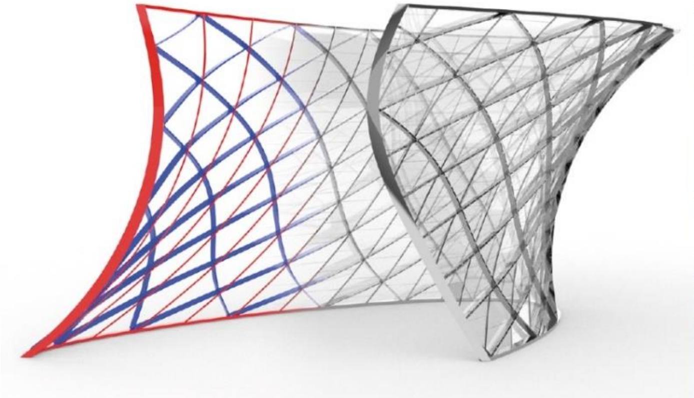
AAG-web



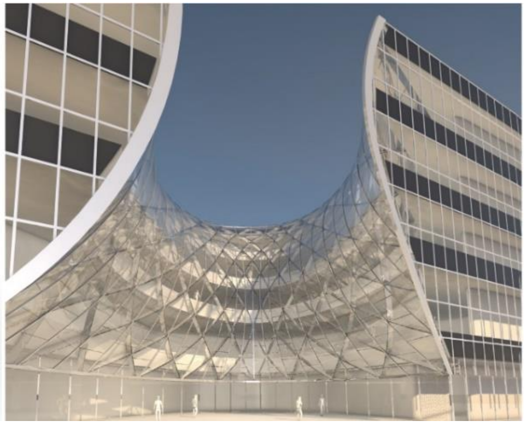
comparison



Emporia shopping center façade



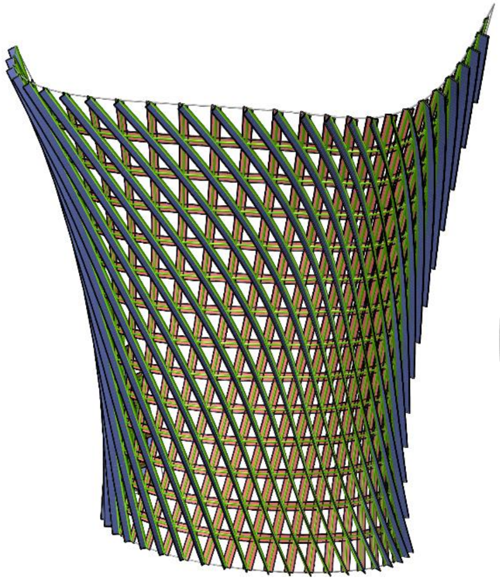
AAG gridshell



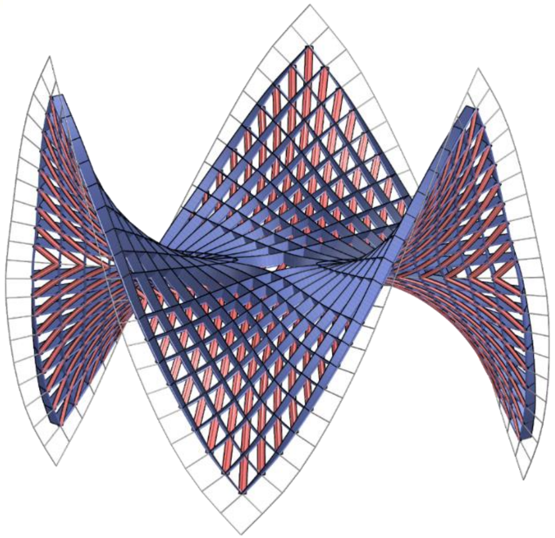
rendering

real model

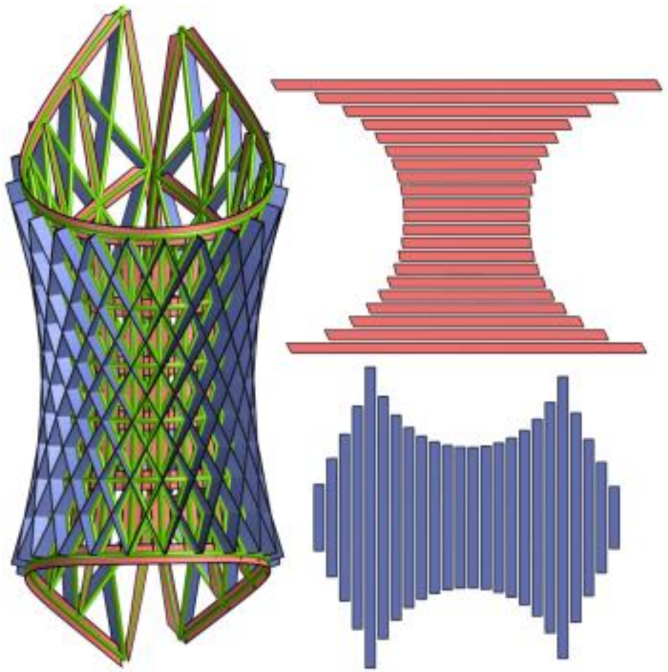
Conclusion



AGG-web



AAG-web



AGAG-web and unrolled strips

AAG timber model

Thank you!