SIGGRAPH ASIA 2023 SYDNEY

# Rectifying Strip Patterns


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Eike Schling[2], Helmut Pottmann[1]

1 KAUST
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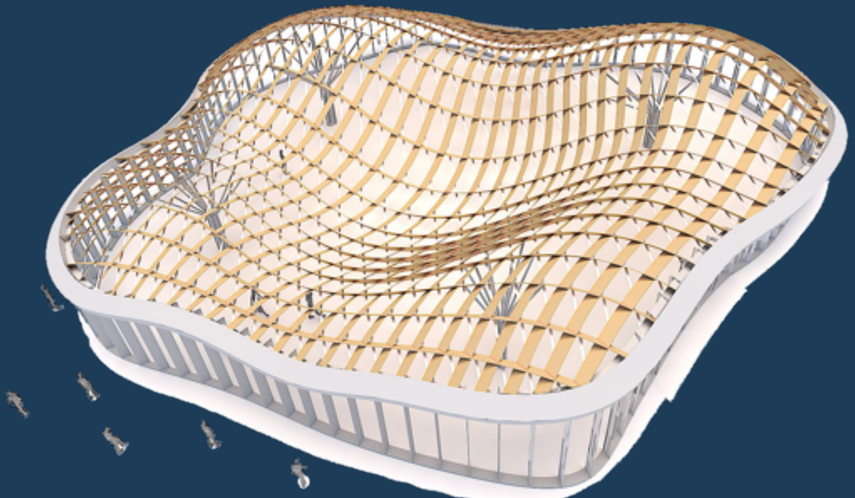
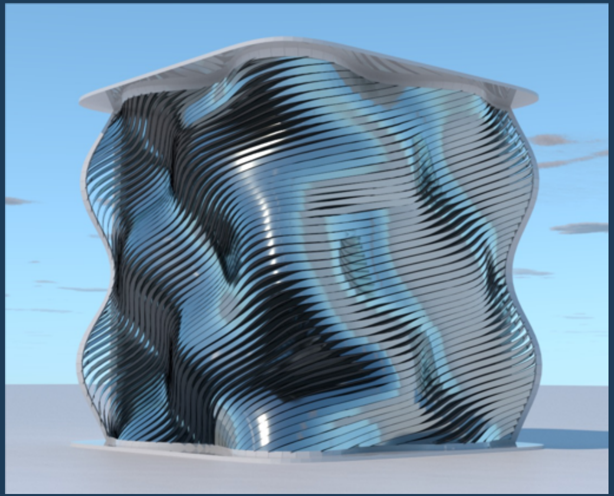
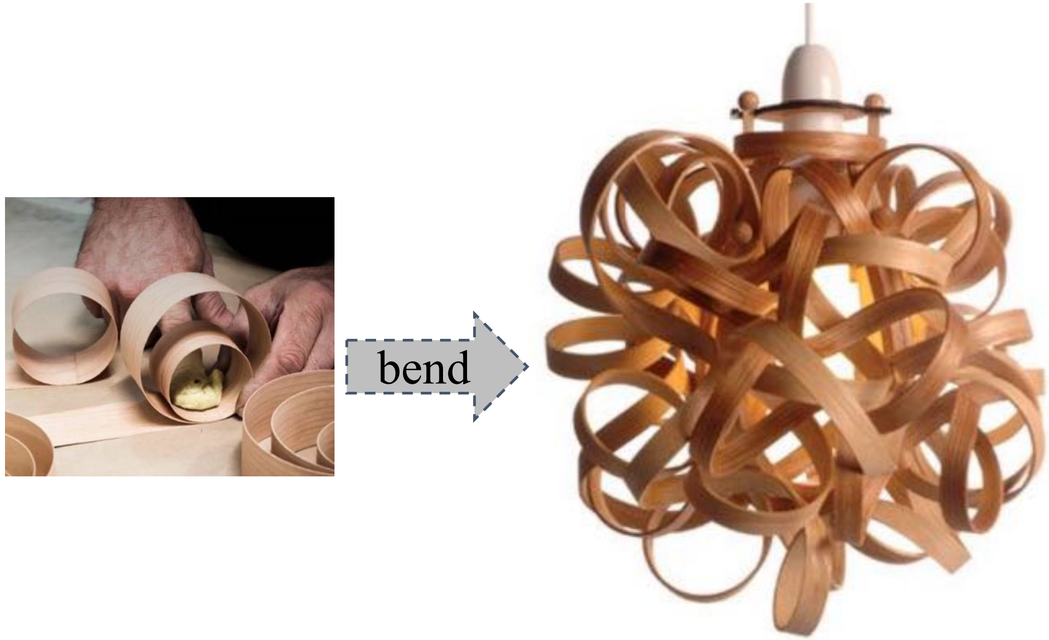


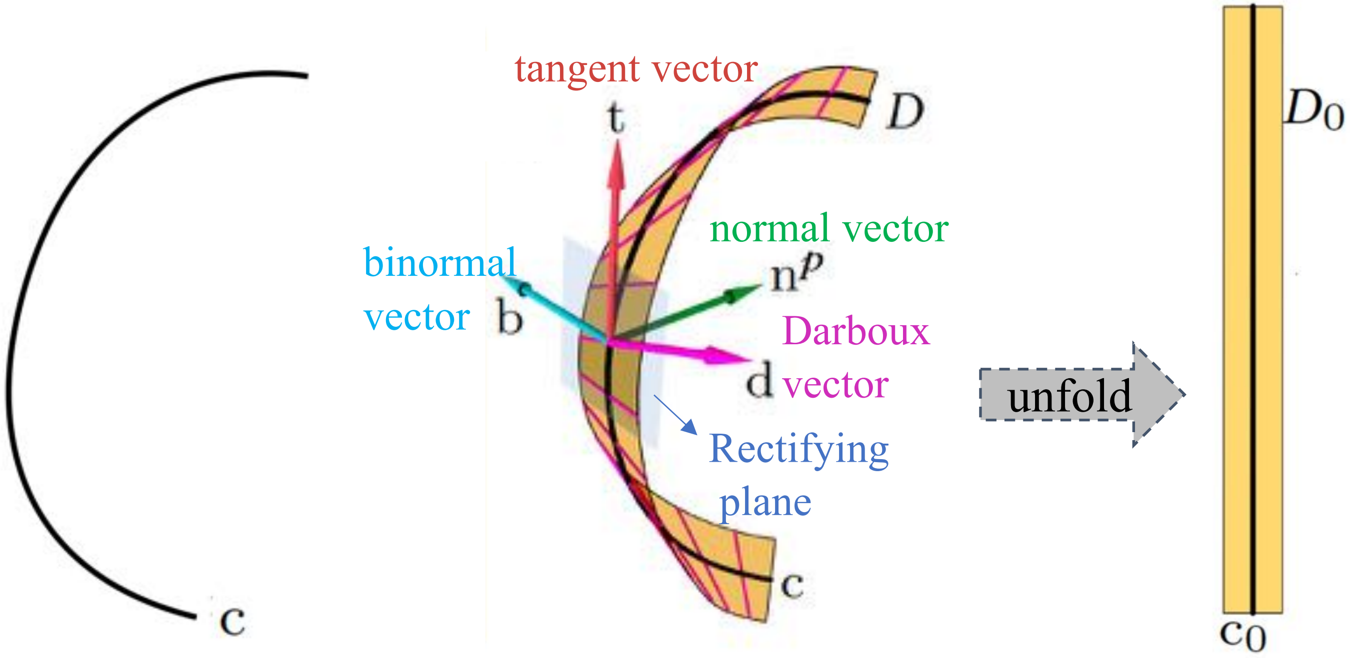


Project Page

# Motivation

# Rectifying Strips in Differential Geometry



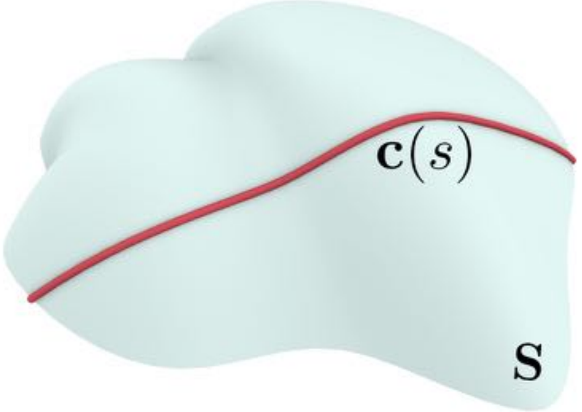



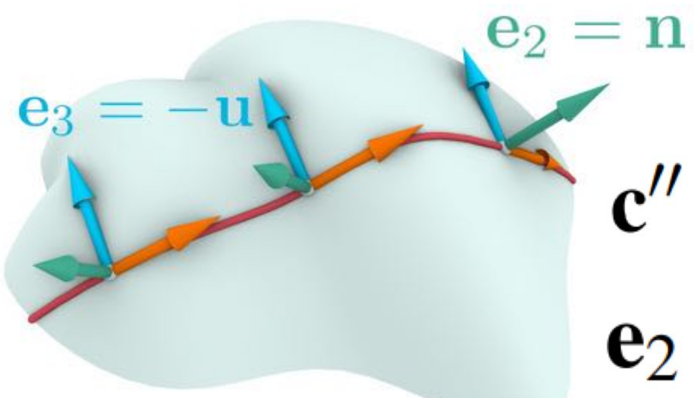

Arbitrary Curve

Rectifying Strip

Straight Flat Strip

# Attaching Rectifying Strips on the Surface



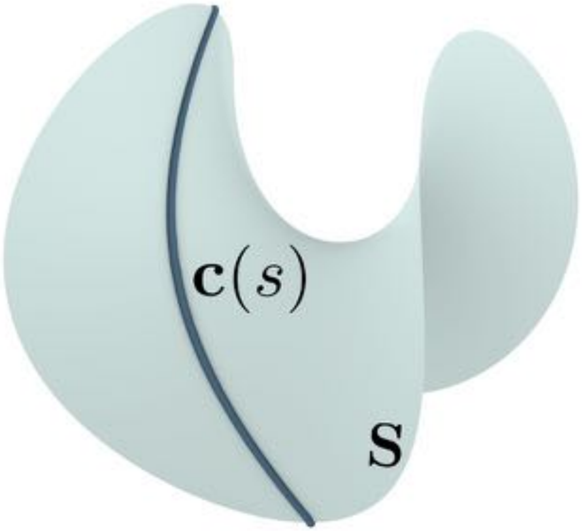



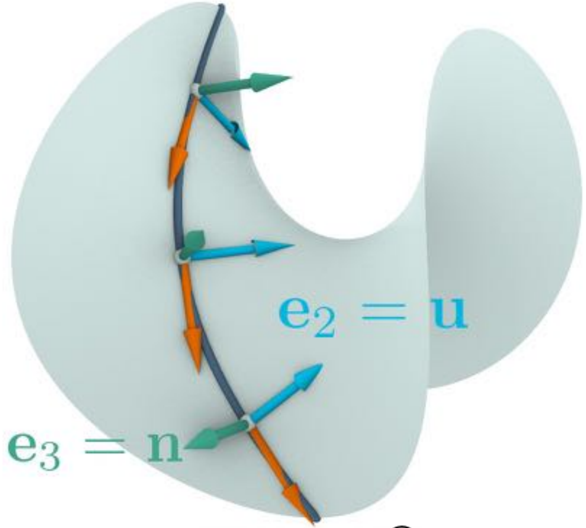

$$\kappa_g = 0$$

$$\mathbf{c}'' = \kappa_n \mathbf{n}$$

$$\mathbf{e}_2 = \mathbf{n}, \mathbf{e}_3 = -\mathbf{u}$$

Geodesic strip

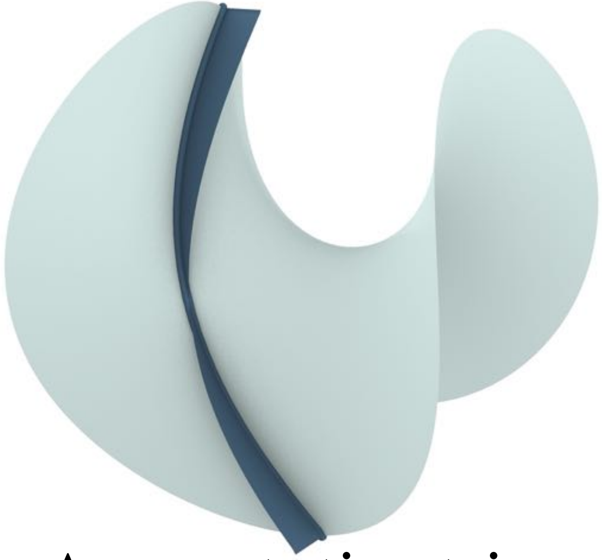



$$\kappa_n = 0$$

$$\mathbf{c}'' = \kappa_g \mathbf{u}$$

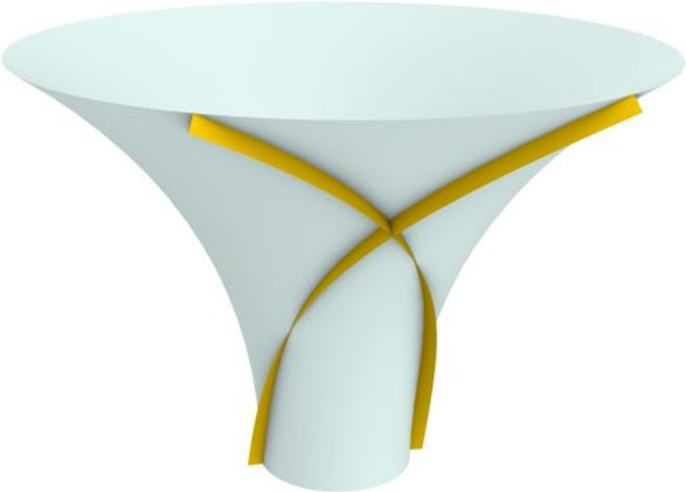
$$\mathbf{e}_2 = \mathbf{u}, \mathbf{e}_3 = \mathbf{n}$$



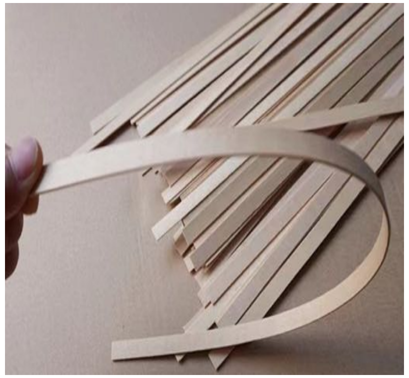
Asymptotic strip

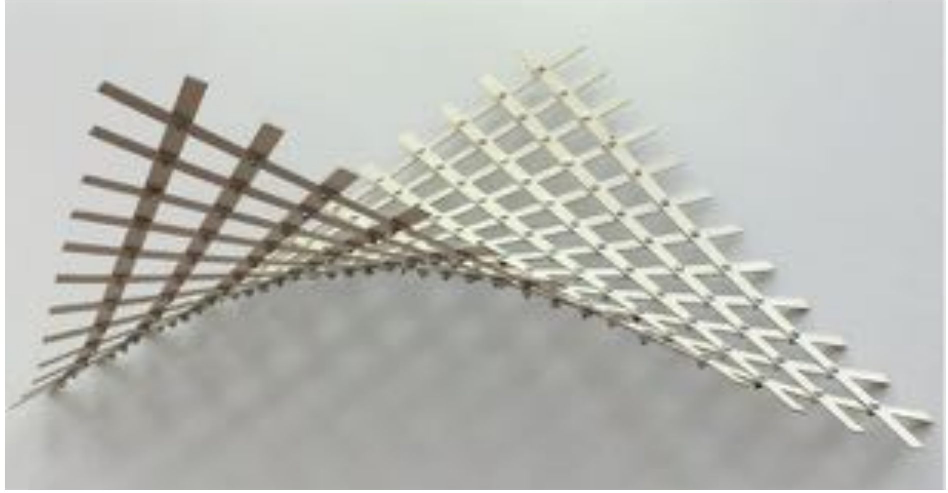
# Motivation: Gridshell Structures

Geodesic strips



Asymptotic strips



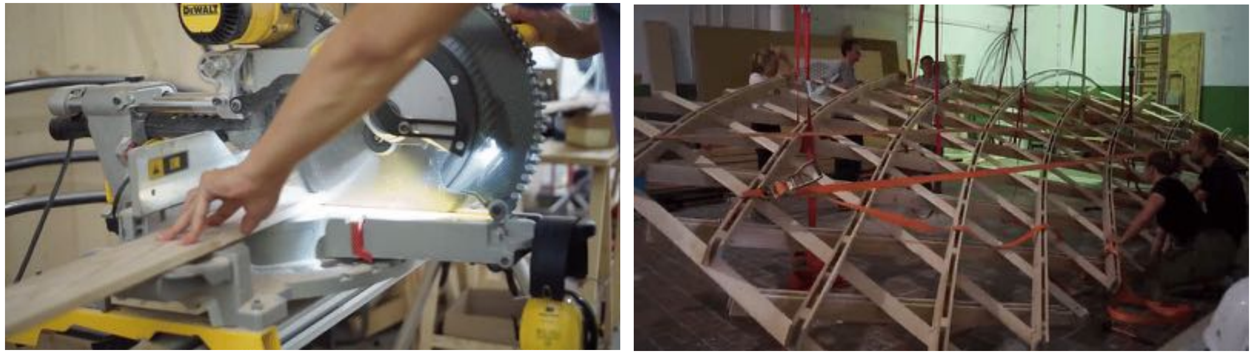


Geodesic gridshell

Asymptotic gridshell

# Motivation: Gridshell Structures

**Fabrication**



**Formation**

**Transportion**

**Fabrication:** Simplification of construction elements, massively produced

**Formation:** bending originally flat straight strips

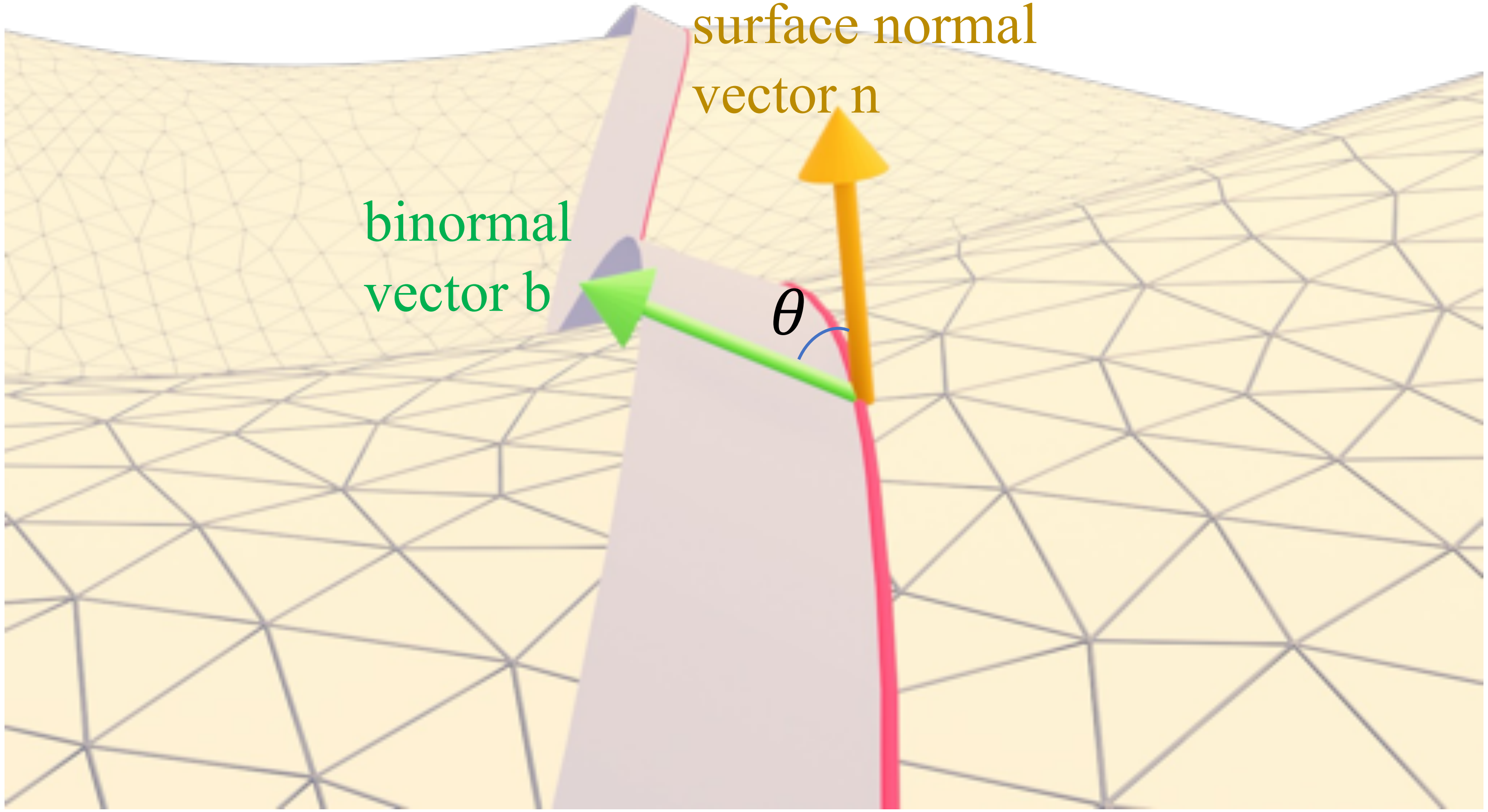
**Transportion:** easily move

*Eike Schling, **Hui Wang**, Sebastian Hoyer, Helmut Pottmann. "Designing asymptotic geodesic hybrid gridshells." Computer-Aided Design 152 (2022): 103378.*
*Eike Schling, Zongshuai Wan, **Hui Wang**, Pierluigi Dacunto. "Asymptotic Geodesic Hybrid Timber Gridshell." Proceedings of AAG, Stuttgart, Germany (2023): 1-12.*

# Motivation: Gridshell Structures

**Pseudo-geodesic curves**:

The signed angle $\theta$ between b and n is constant. [*W. Wunderlich, 1950*]

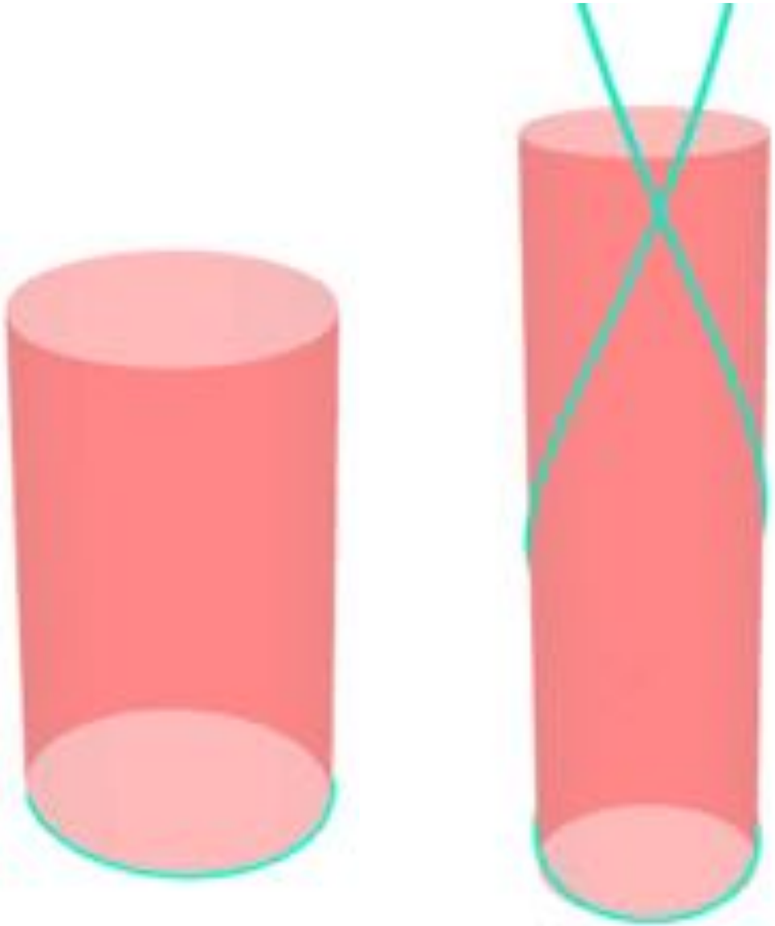



$\theta = 90°$, geodesic

$\theta = 0°$, asymptotic

# Motivation: Gridshell Structures

$$\begin{cases} x = a\cos t \\ y = a\sin t \\ z = a\tan\theta\,\mathrm{ch}\left(\dfrac{t}{\tan\theta}\right) \end{cases}$$



Parametrization | Given t | Changing t

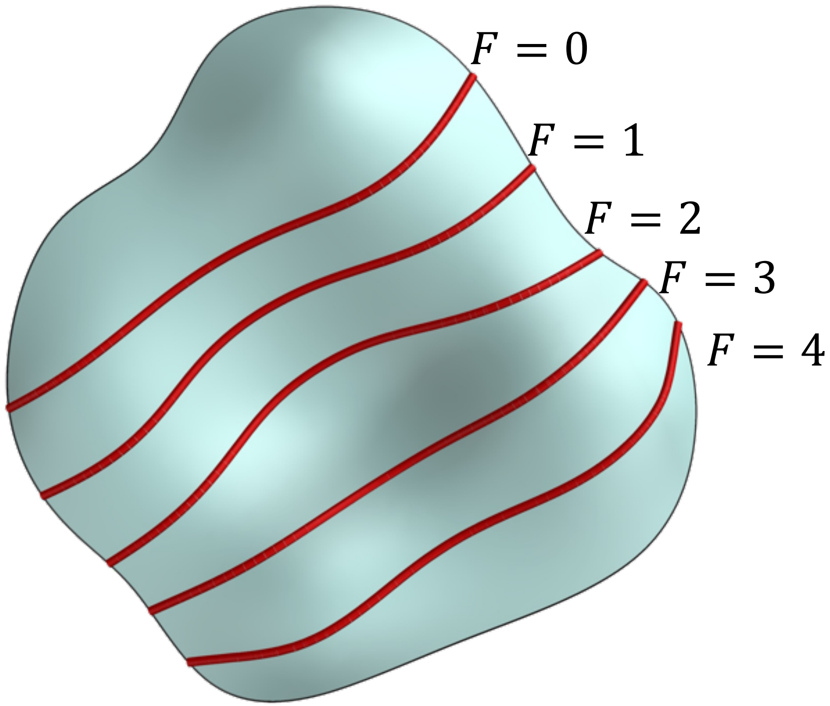
Pseudo-geodesic curve on a cylinder


Strips holding a constant angle to the surface
*[Mesnil and Baverel 2023]*

# Method and Results

# Method: A Level-Set Based Framework

## Assign function values for the curves




A robust version of
[Jiang et al. 2019]'s tracing algorithm.

## Optimize the level sets to fit the curves

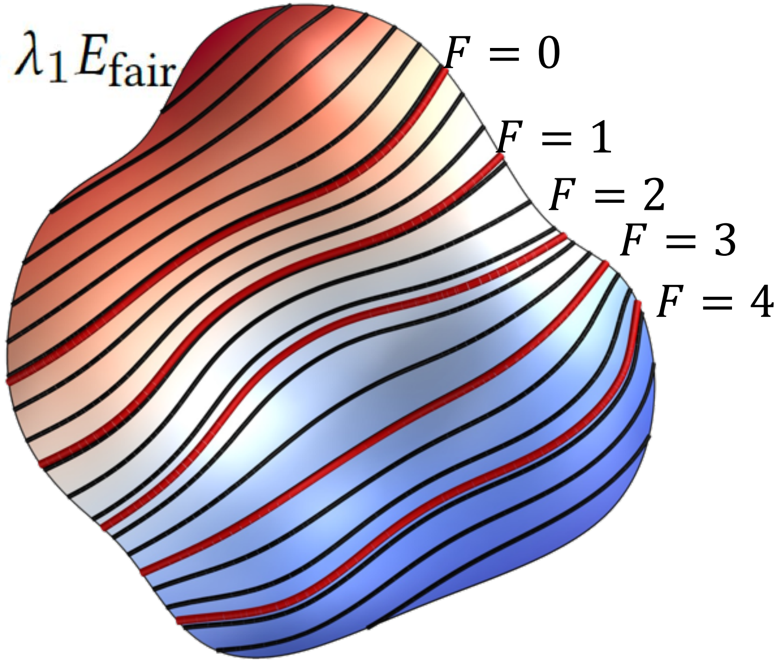
Linear interpolation:

$$E_{\text{trace}} = \sum_{c^i \in C_t} \sum_{p \in c^i} \left( \frac{F^i - F_0}{F_1 - F_0} - \frac{\|p - v_0\|}{\|v_1 - v_0\|} \right)^2.$$

Fairness:

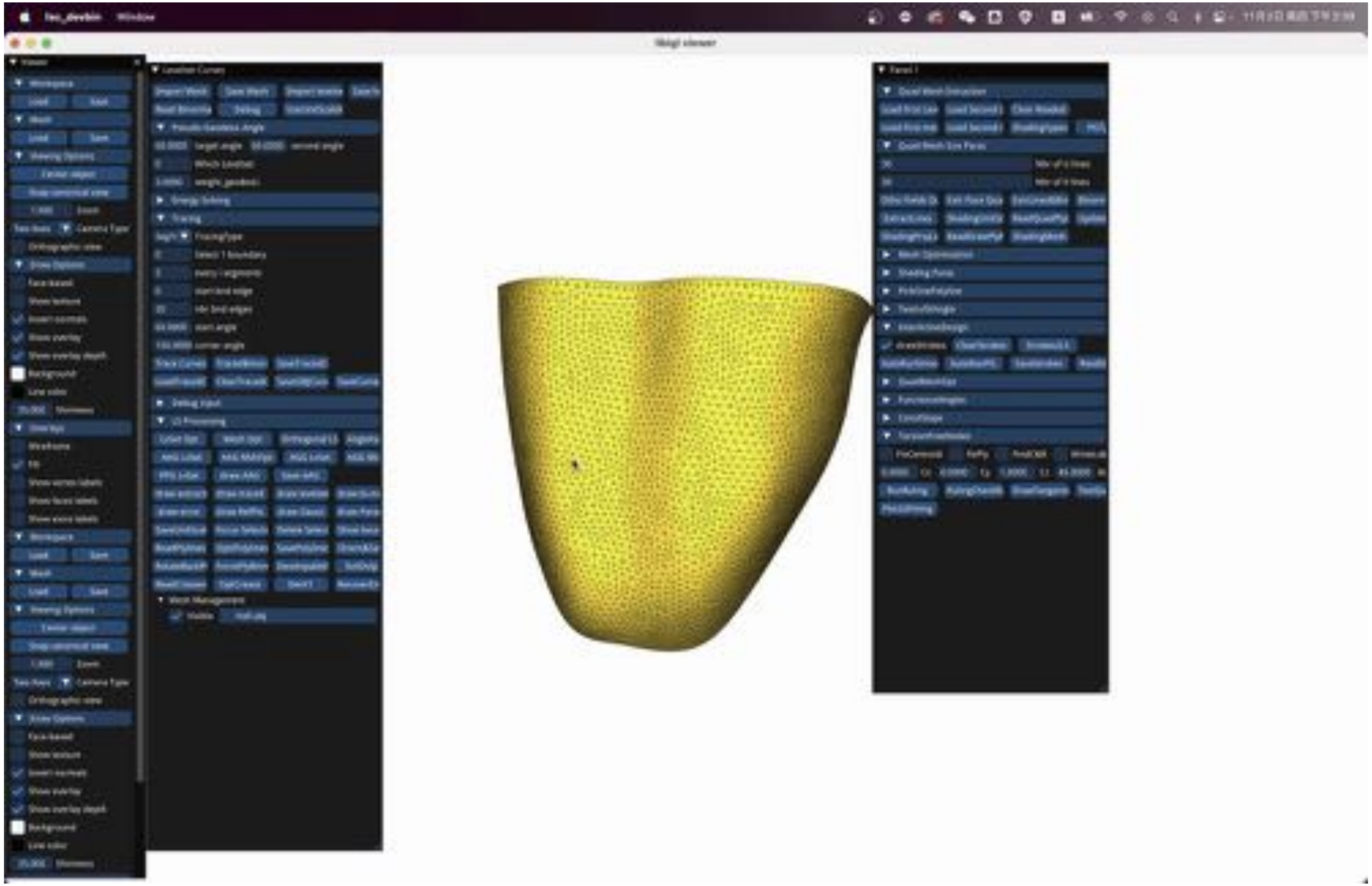
$$E_{\text{fair}} = \sum_{v \in \mathcal{V}} \|H(v)\|^2 \mathcal{A}(v).$$

$$\min \quad E_{\text{init}} = \lambda_0 E_{\text{trace}} + \lambda_1 E_{\text{fair}}$$



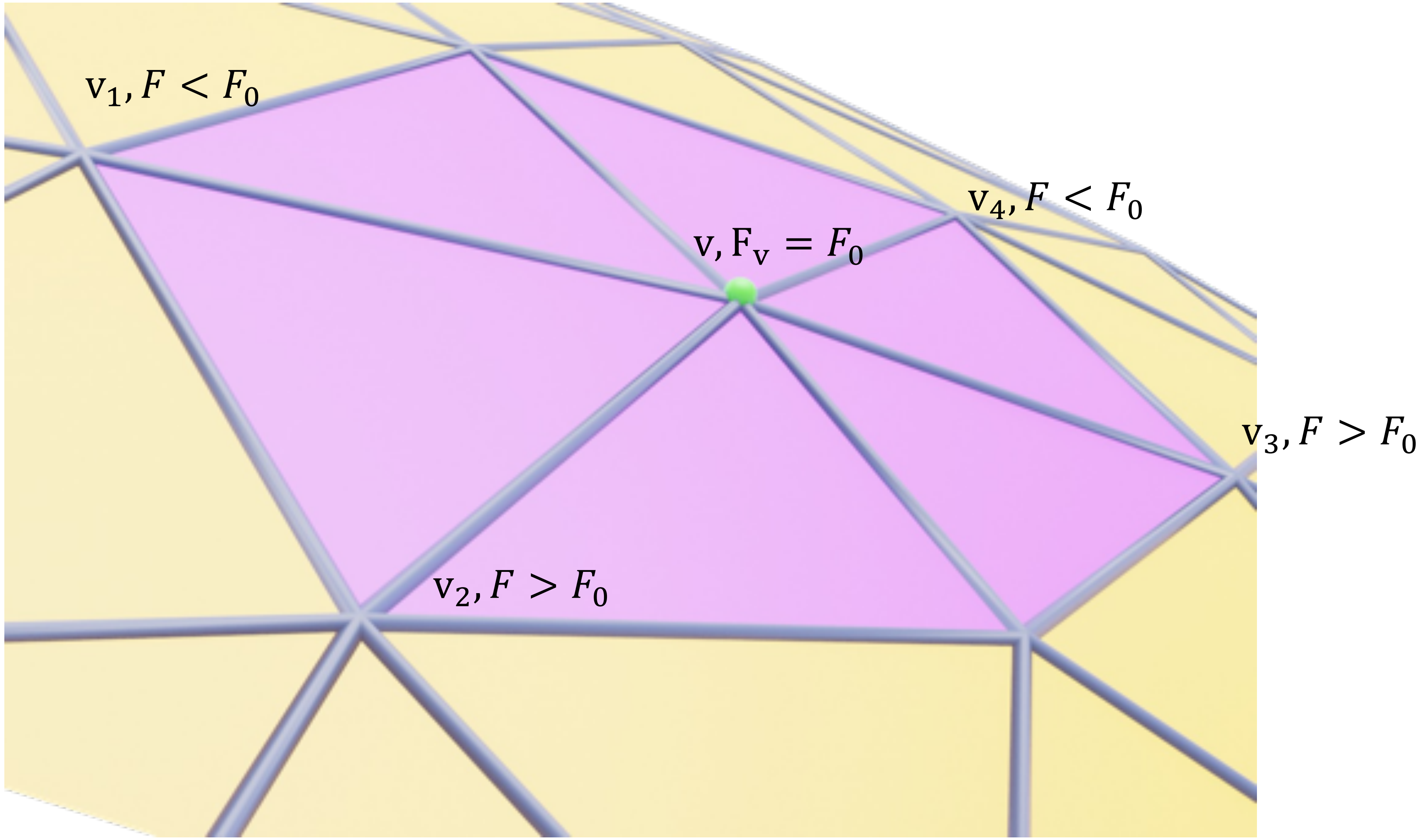
# Method: A Level-Set Based Framework

An interactive method (optional initialization):



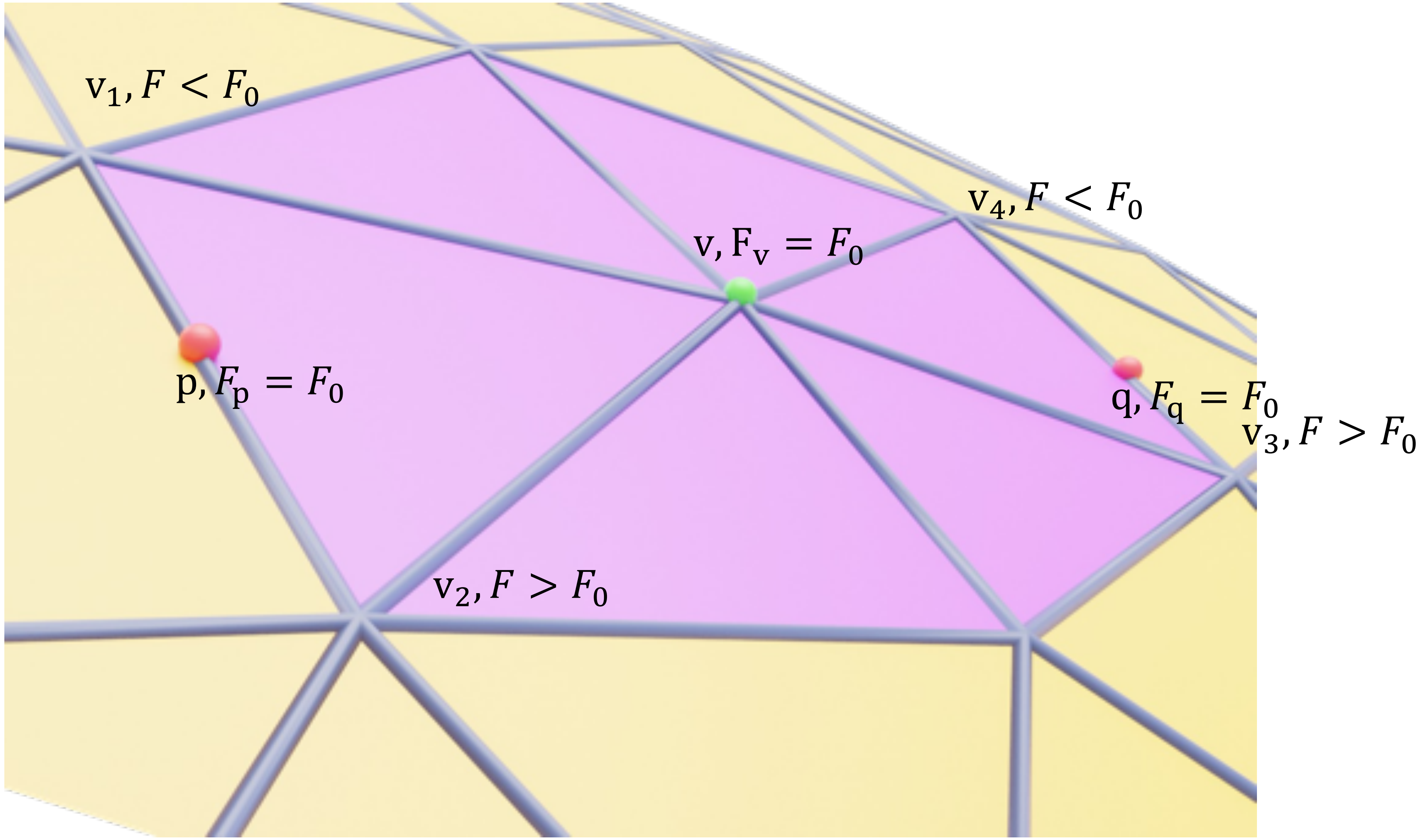
# Optimizing Pseudo-geodesics

Control the inclination angles ←→ Controlling the binormals on each vertex star



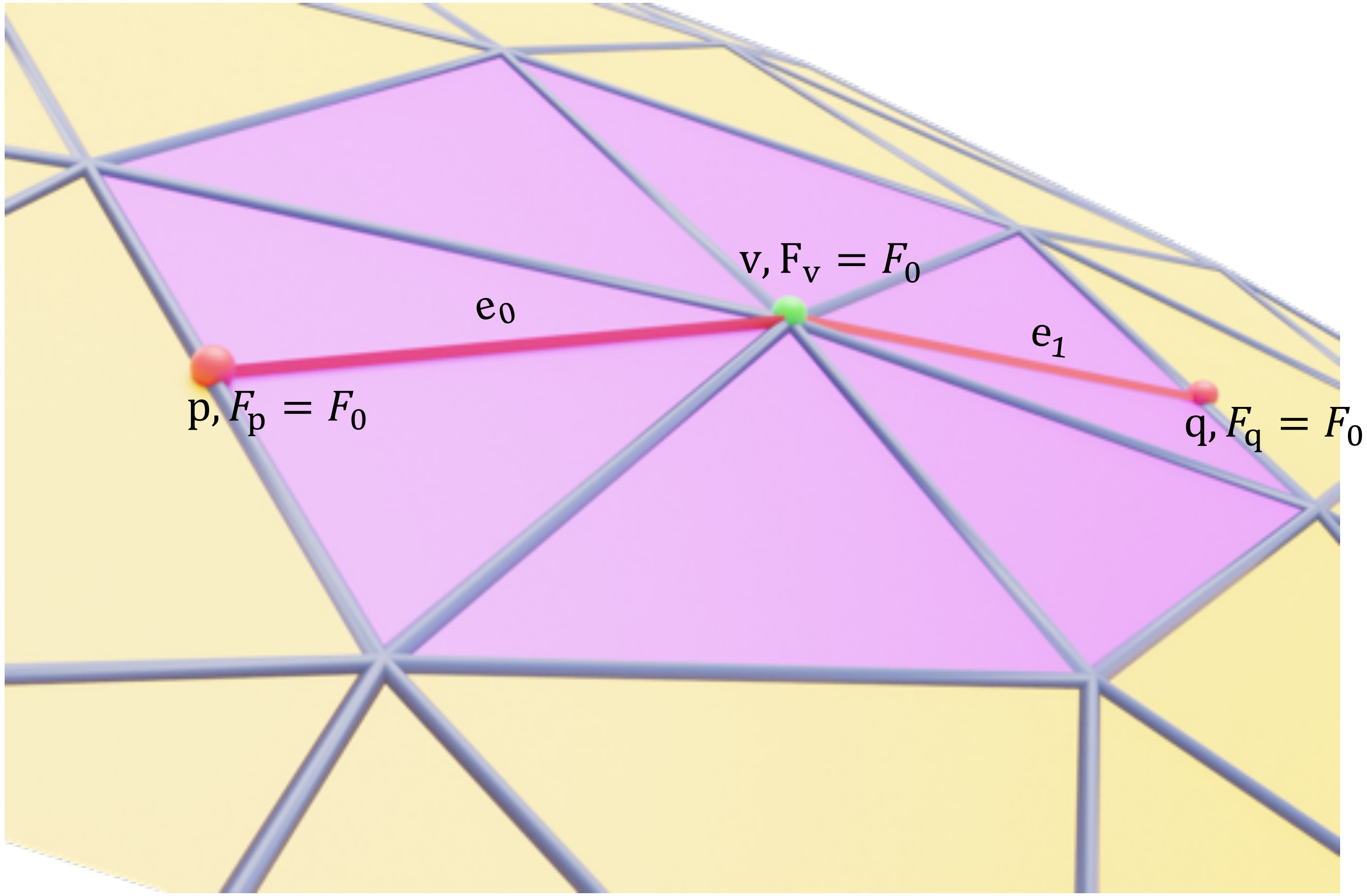
# Optimizing Pseudo-geodesics

Control the inclination angles ←→ Controlling the binormals on each vertex star



# Optimizing Pseudo-geodesics

Control the inclination angles ←→ Controlling the binormals on each vertex star

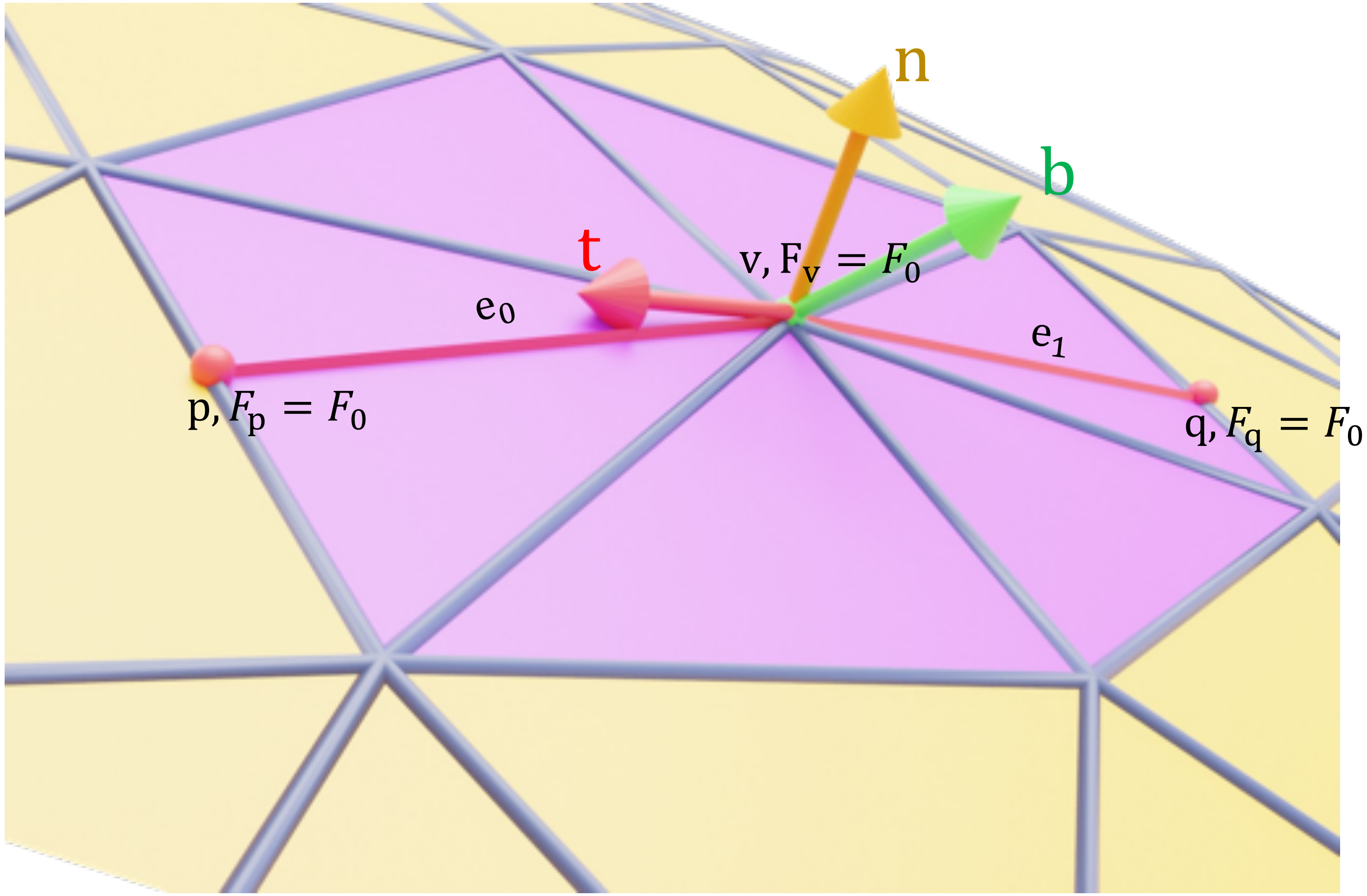



$$e_0 = \frac{p - v}{||p - v||},$$

$$e_1 = \frac{v - q}{||v - q||},$$

# Optimizing Pseudo-geodesics

Control the inclination angles ←→ Controlling the binormals on each vertex star




$$e_0 = \frac{p - v}{||p - v||},$$

$$e_1 = \frac{v - q}{||v - q||},$$

$$b = \frac{e_0 \times e_1}{||e_0 \times e_1||}$$

Angle constraint:

$$\min\,(b \cdot n - \cos\theta)^2$$

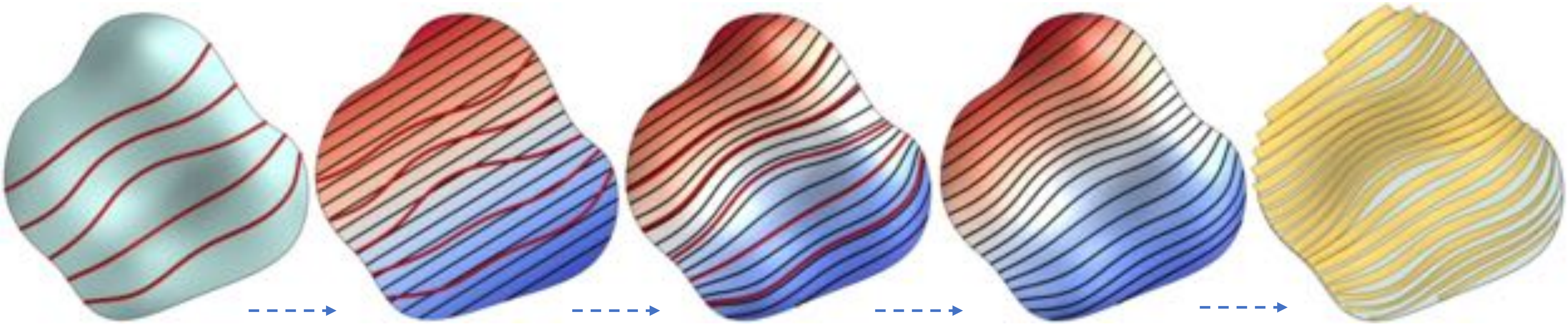
# Optimizing Pseudo-geodesics

Angle constraints: $E_{\text{angle}} = \sum_{\mathrm{v} \in \mathcal{V}} ((\mathrm{b} \cdot \mathrm{n})^2 - \cos^2\theta)^2 \mathcal{A}(\mathrm{v}) + \sum_{\mathrm{v} \in \mathcal{V}} ((\mathrm{b} \cdot \mathrm{n})(\mathrm{b} \cdot \mathrm{u}) - \sin\theta\cos\theta)^2 \mathcal{A}(\mathrm{v}),$

Preventing vanishing gradients: $E_{\text{grad}} = \sum_{\mathrm{f} \in \mathcal{F}} (\|\nabla F(\mathrm{f})\| - r)^2 \mathcal{A}(\mathrm{f}),$

Fairness: $E_{\text{fair}} = \sum_{\mathrm{v} \in \mathcal{V}} \|H(\mathrm{v})\|^2 \mathcal{A}(\mathrm{v}).$

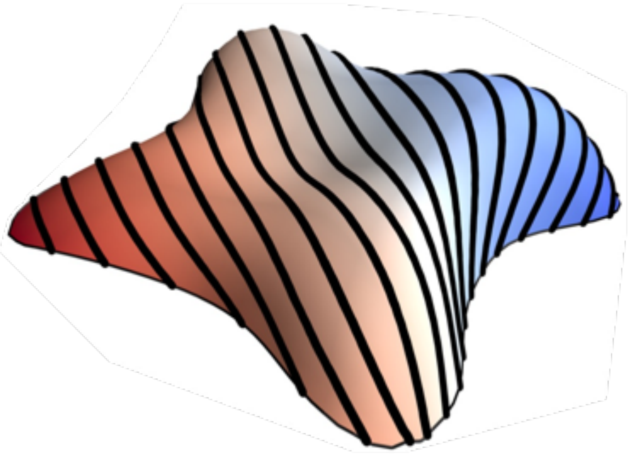
min $E_{\text{pg}} = \lambda_{\text{fair}} E_{\text{fair}} + \lambda_{\text{grad}} E_{\text{grad}} + \lambda_{\text{angle}} E_{\text{angle}}.$



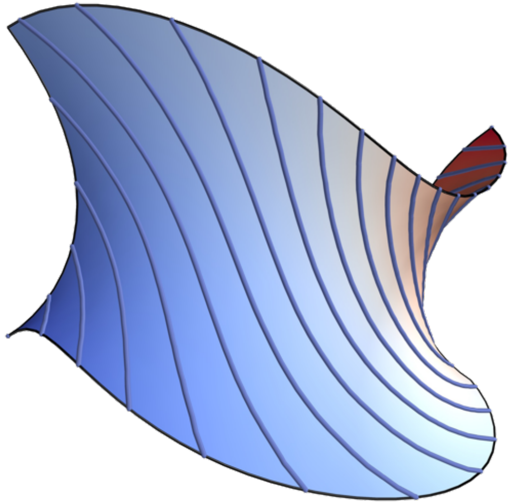
Input surface + target angle $\theta$

Pseudo-geodesics of angle $\theta$

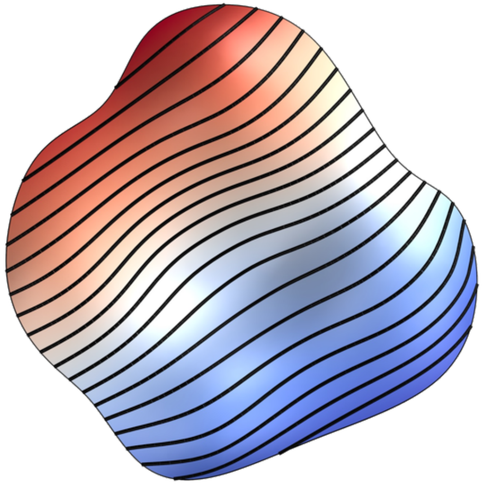
# Optimizing 1-family of Pseudo-geodesics



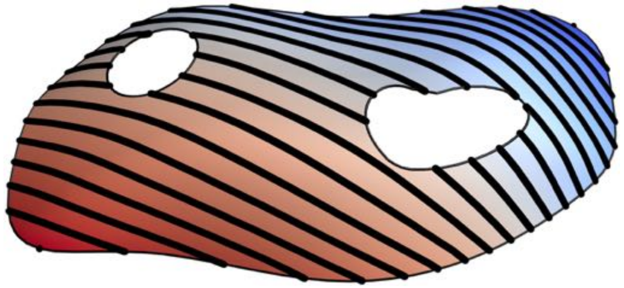
$\theta = 90°$
Geodesic



$\theta = 0°$
Asymptotic



$\theta = 60°$
Pseudo-geodesic

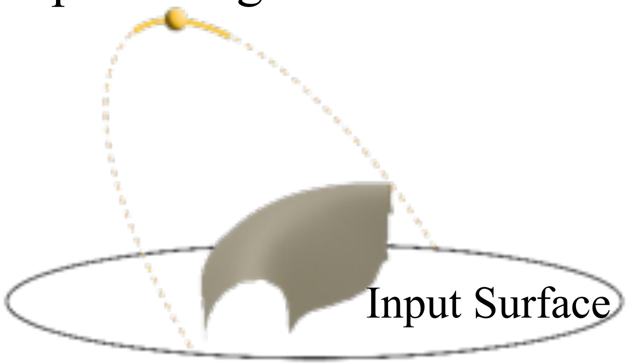


$\theta = 75°$
Pseudo-geodesic

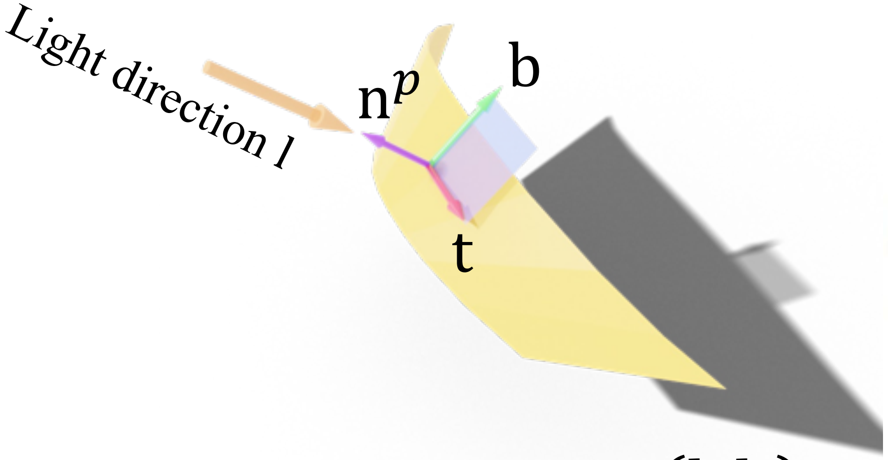
# Applications: Shading Systems

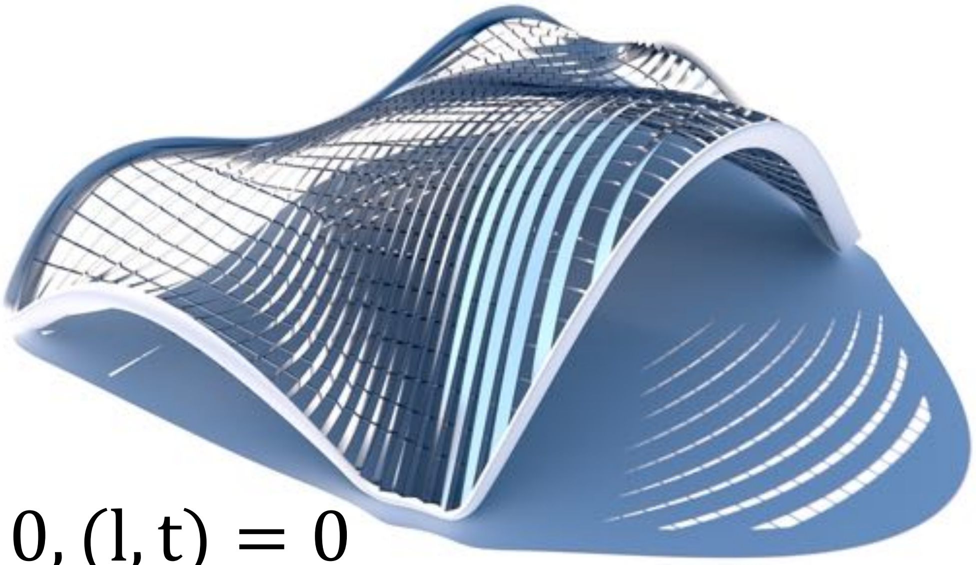
Makkah, 12:00, Dec 1st.

Input sunlight direction

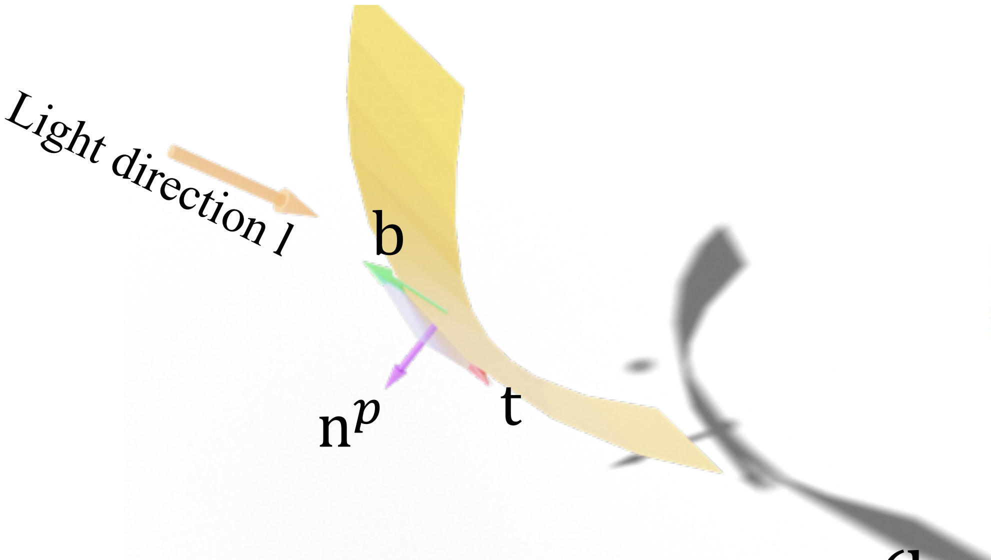


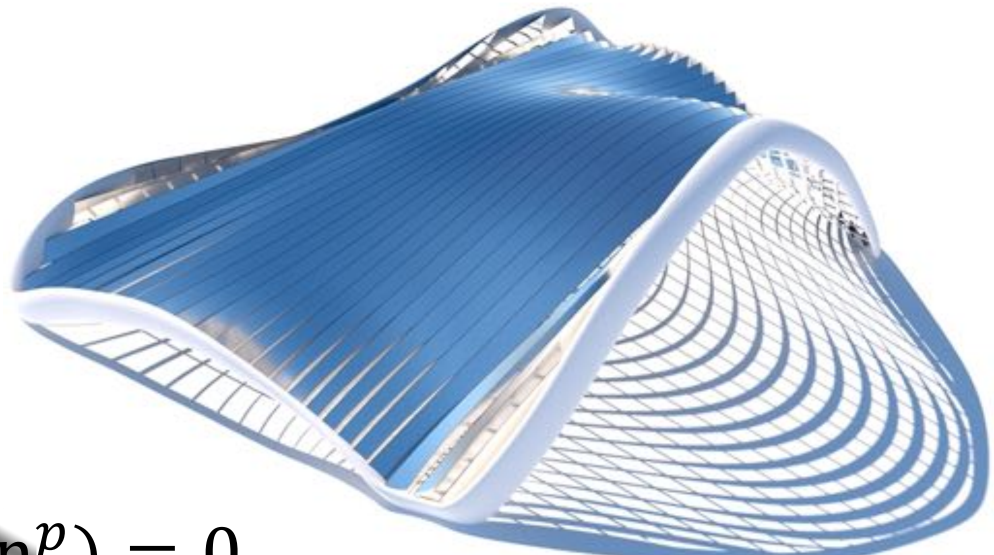

Earch ground



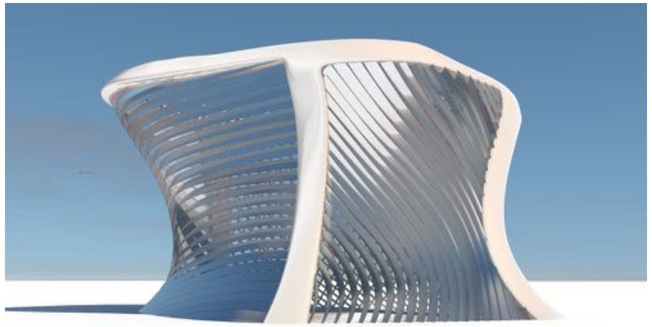
Blocking the light: $(l, b) = 0, (l, t) = 0$



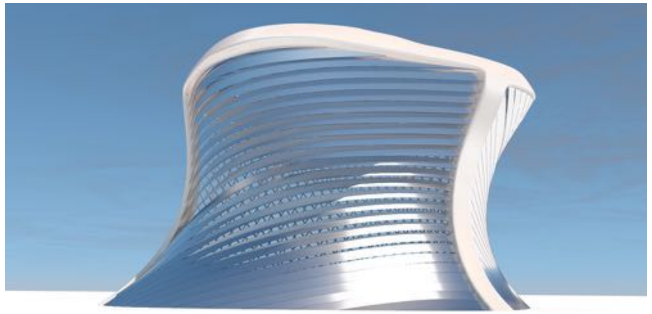



Letting the light through: $(l,\ n^p) = 0$

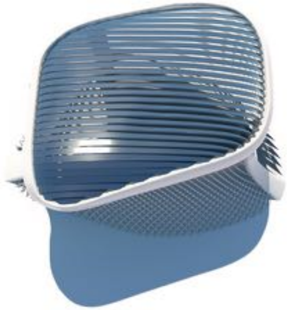
# Applications: Shading Systems



Light through

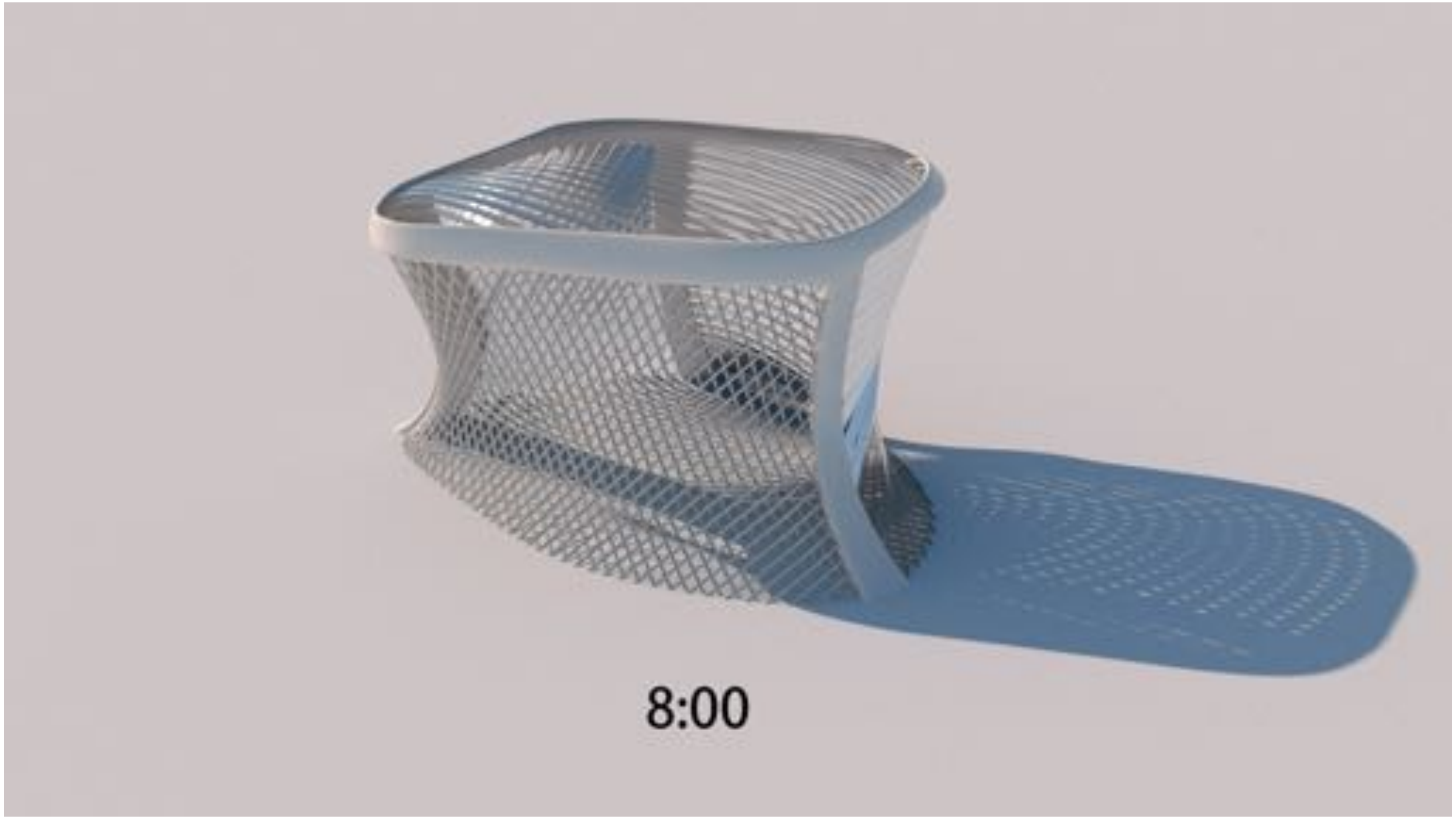


Light blocked



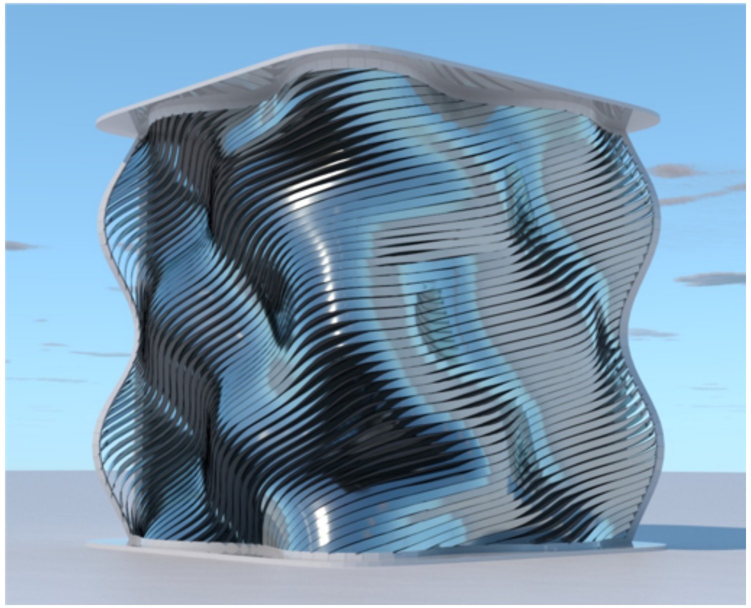
Light blocked

Vienna, Aug 1st



# Applications: Shading Systems

Sunlight through in the morning,
and sunlight blocked in the afternoon.

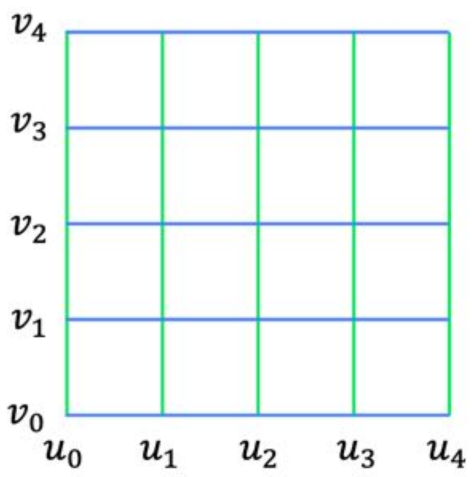


Outside view

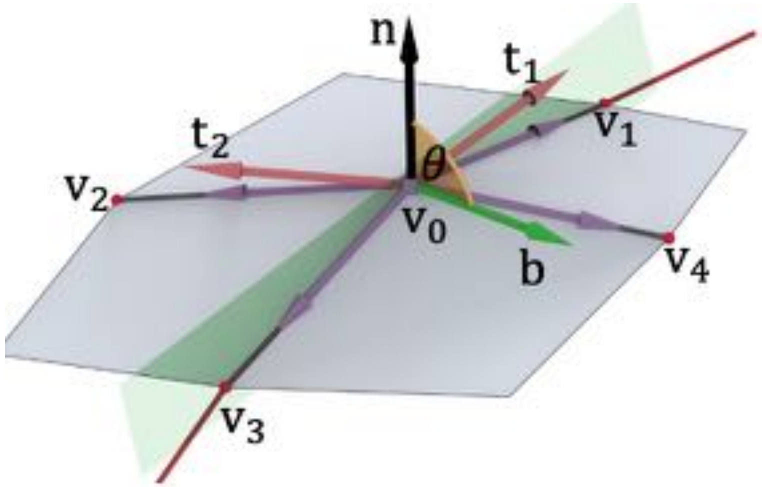
London, Aug 15th


Inside view

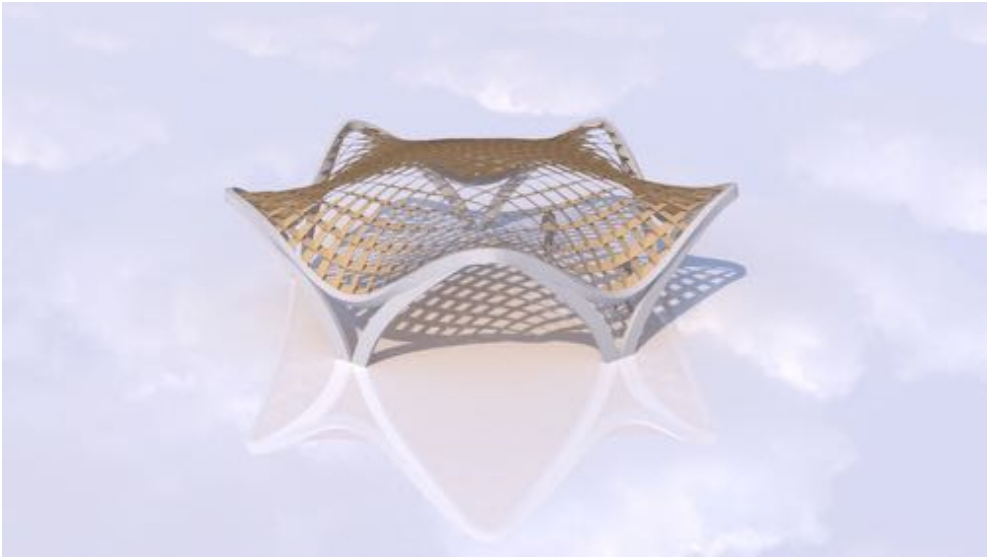
# Optimizing 2-family of Pseudo-geodesics




Parametric net



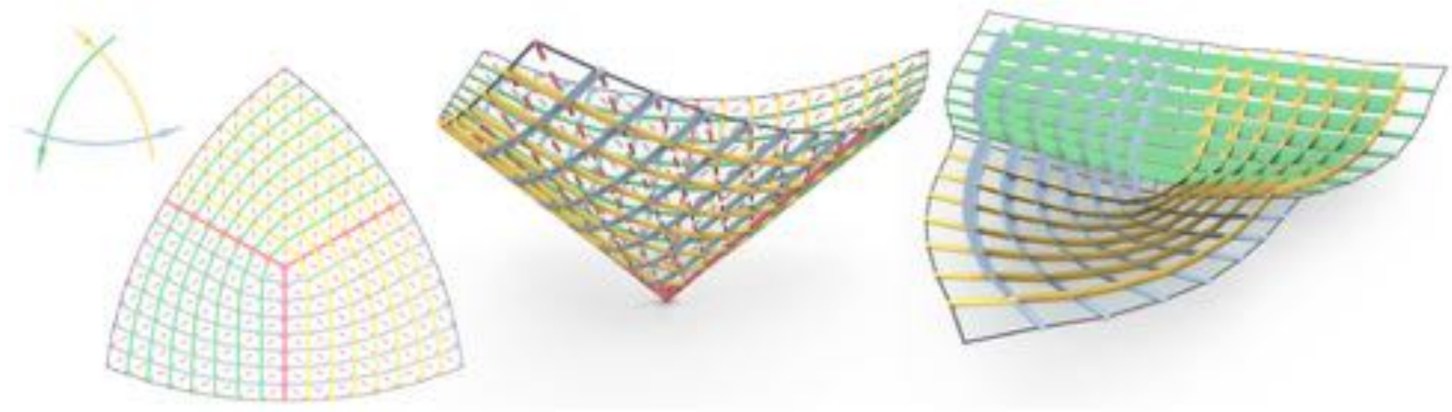

Vertex star



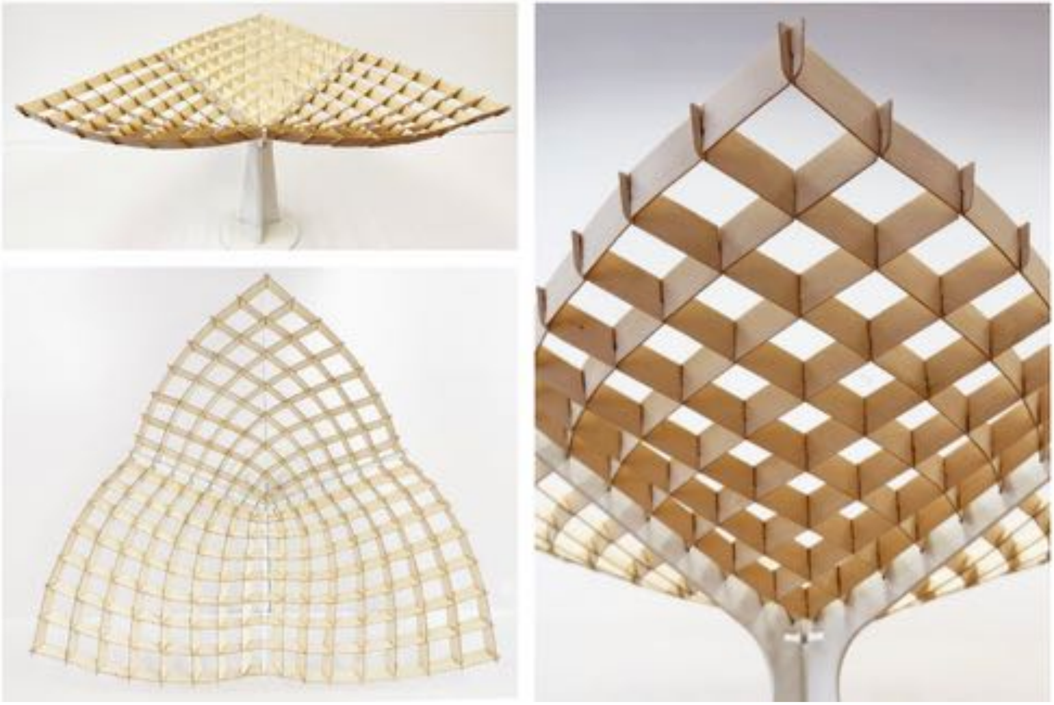
Pseudo-geodesic net (PP-net) with
$\theta_1 = \theta_2 = 60°$

$$E_{\text{angle}} = \sum_{v \in \mathcal{V}} ((b \cdot n)^2 - \cos^2\theta)^2 + \sum_{v \in \mathcal{V}} ((b \cdot n)(b \cdot u) - \sin\theta\cos\theta)^2$$

# Applications: Gridshell Structures

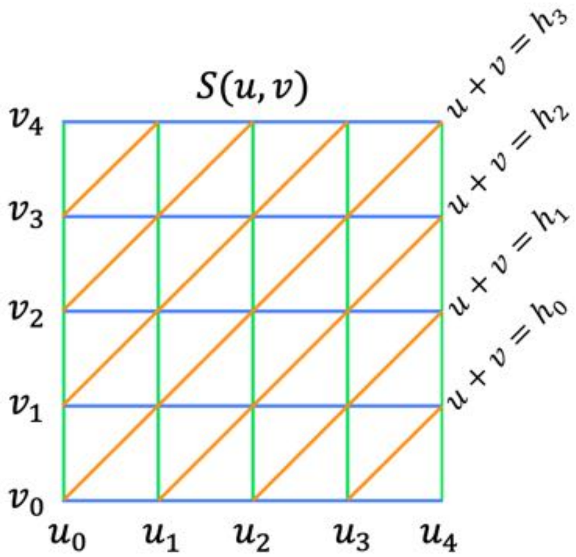


PP-net
$\theta_1 = \theta_2 = 50°$

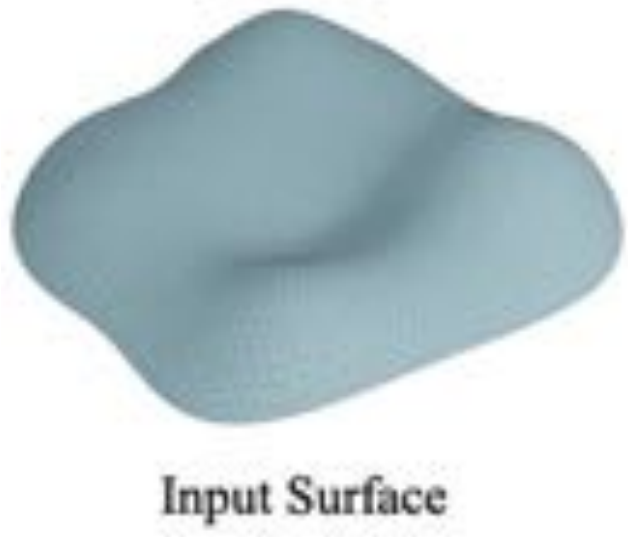


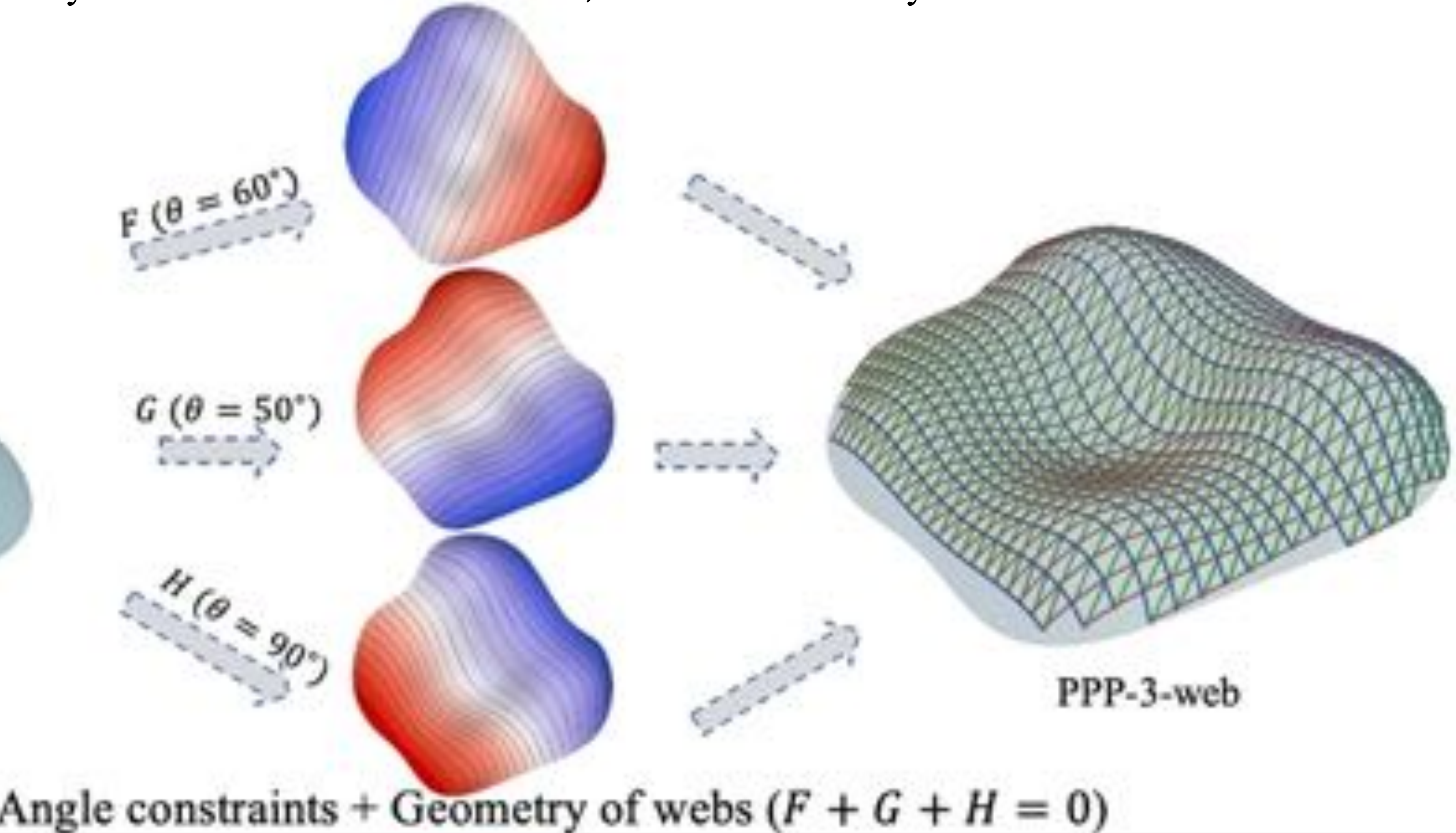
Physical model

# Optimizing 3-family of Pseudo-geodesics

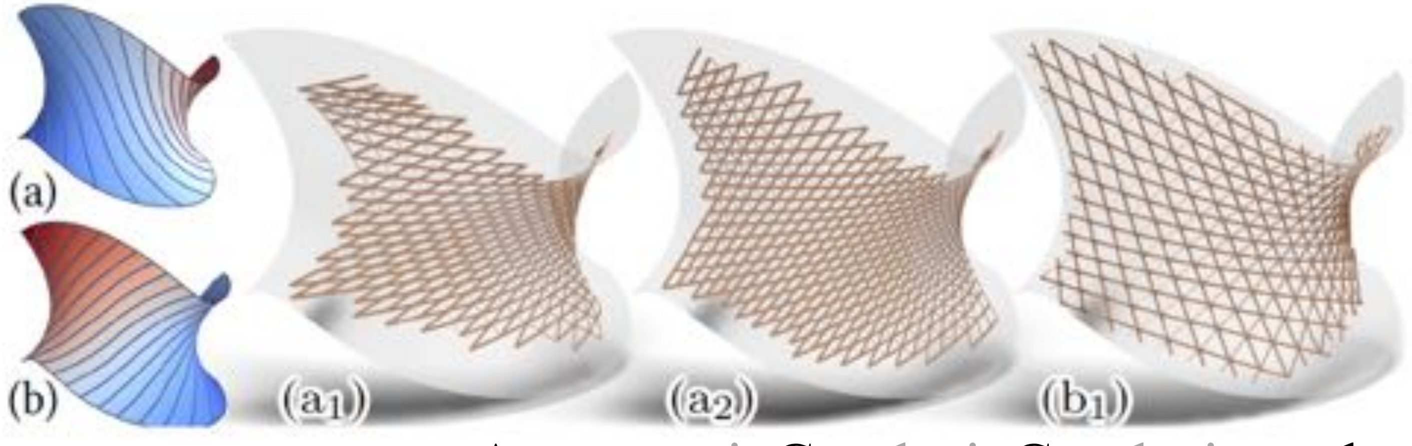



Geometry of webs *[Blaschke and Bol 1938]:* The 3-web on $S$ can be represented by the level-sets of 3 functions $F, G$ and $H$ that satisfy $F+G+H=0$

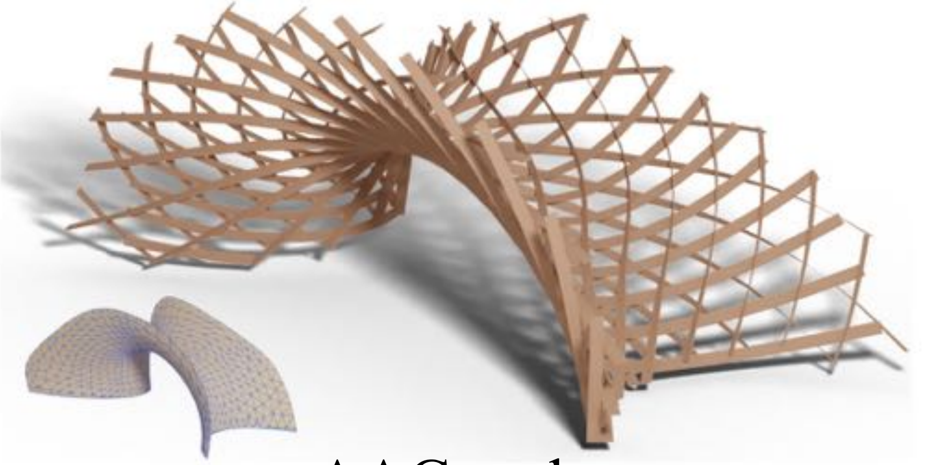


# Applications: Gridshell Structures




AsymptoticGeodesicGeodesic-web



AAG-web

PPG-web $\theta_1 = 30°, \theta_2 = 45°$

Physical model: PPG-web $\theta_1 = \theta_2 = 60°$

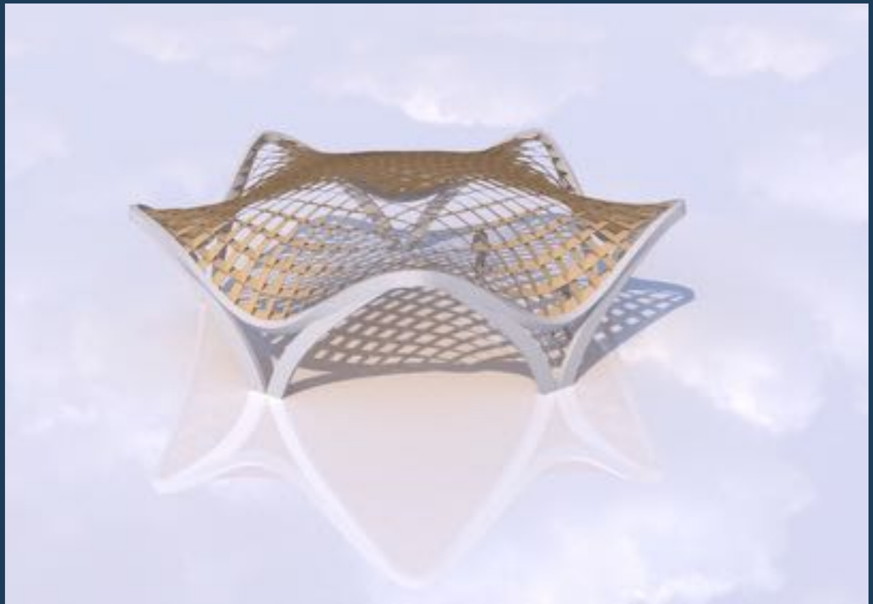
# Contribution and Conclusion

Github Code

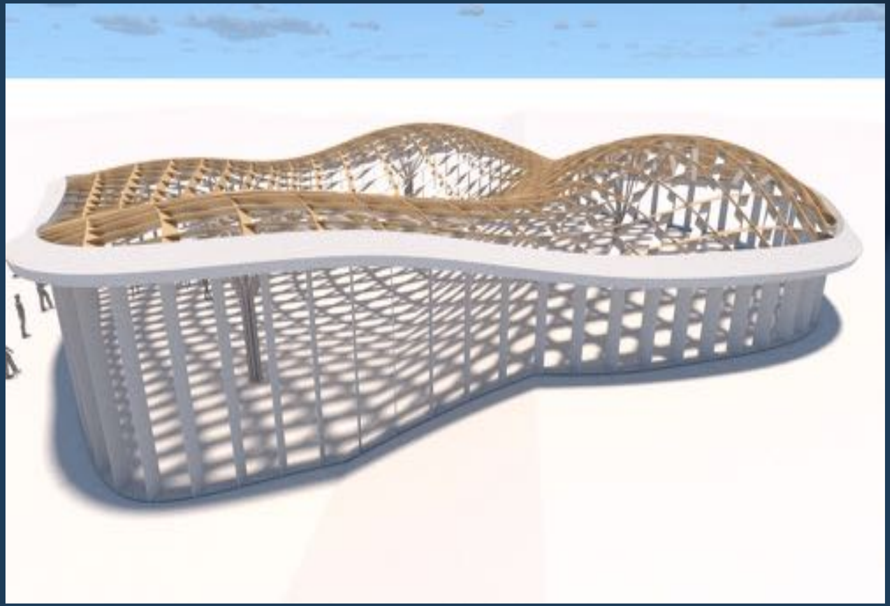
Project Page

- Computational design of shapes from rectifying strips (straight flat strips)
- Controllable inclinations of rectifying strips along level-set curves
- Various rectifying strip patterns applied in shading system and gridshell structures

1-family
Shading system



2-family
PP-gridshell



3-family
PPG-gridshell